# Online EM algorithm to solve the SLAM problem

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## Introduction

A new algorithm - the *onlineEM-SLAM* - is proposed to solve the simultaneous localization and mapping problem (SLAM). The mapping problem is seen as an instance of inference in latent models, and the localization part is dealt with a particle approximation method. This new technique relies on an online version of the Expectation Maximization (EM) algorithm, see [1, 2].

## Model

The robot evolves in a 2-dimensional landmark-based map. Let $\boldsymbol{\theta}$ be all the landmarks in the map and $\mathbf{x_t} = (x_{t,1}, x_{t,2}, x_{t,3})^T$ be the robot pose. Controls are denoted by $\mathbf{u_t} = (v_t, \psi_t)^T$ where $\psi_t$ stands for the robot's heading direction and $v_t$ its velocity. The underlying model is given by

- $\mathbf{x_t} = f(\mathbf{x_{t-1}}, \hat{u}_t)$, where $\hat{u}_t \sim \mathcal{N}(\mathbf{u_t}, Q)$.

- $\mathbf{y_{t,i}} = h(\mathbf{x_t}, \theta_{.,i}) + \delta_{t,i}$ , for any $i \in \mathcal{A}_t$ (set of observed landmarks at time $t$), where $h$ is defined by

$$h(\mathbf{x}, \tau) = \begin{pmatrix} \sqrt{(\tau_1 - \mathbf{x}_1)^2 + (\tau_2 - \mathbf{x}_2)^2} \\ \arctan \frac{\tau_2 - \mathbf{x}_2}{\tau_1 - \mathbf{x}_1} - \mathbf{x}_3 \end{pmatrix} .$$

  $(\delta_{t,i})_{t, i \in \mathcal{A}_t}$ are i.i.d Gaussian mixtures with components $\mathcal{N}(0, \sigma_0^2 R)$, $\mathcal{N}(0, \sigma_1^2 R)$ and weights $(\omega_0, \omega_1)$.

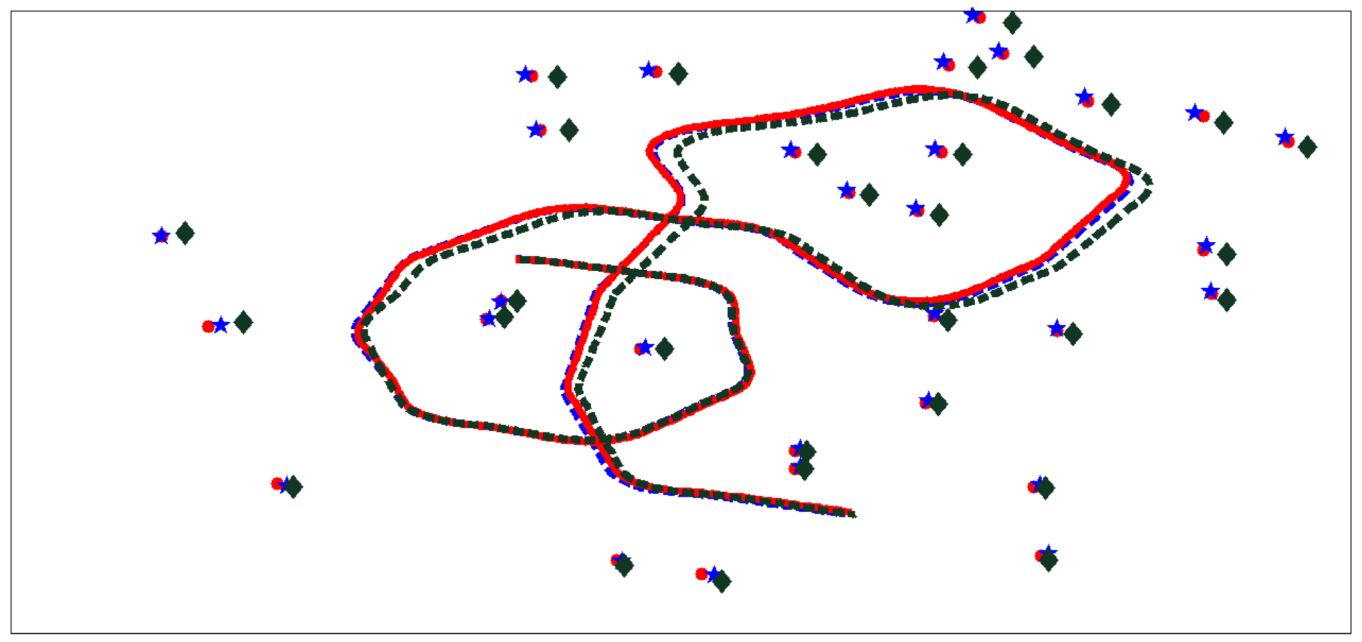
$R$, $Q$, $\sigma_0^2$ and $\sigma_1^2$ are assumed to be known. $\mathbf{I_t} = (I_{t,i})_{i \in \mathcal{A}_t}$ denotes the latent variables specifying the mixture component of each observation at time $t$ and we write $\mathbf{Z_t} = (\mathbf{X_t}, \mathbf{I_t})$ the extended latent variable. The likelihood of $\mathbf{y_t}$ given $\mathbf{z_t}$ is denoted by $g_\theta(\mathbf{y_t}|\mathbf{z_t})$ and the state transition density by $m(\mathbf{x_t}|\mathbf{x_{t-1}}, \mathbf{u_t})$.

## Experimental results

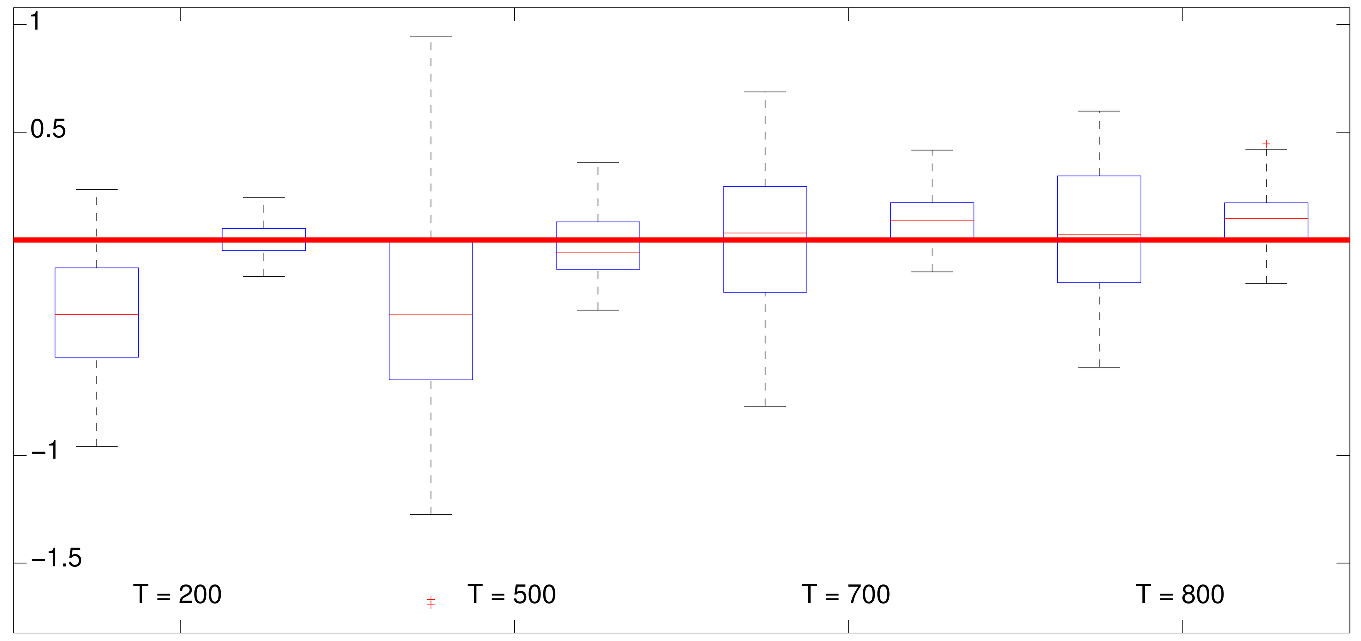
Experimental settings can be found in [3]. This new algorithm is compared to the marginal SLAM (see [4]), the function $f$ is given by

$$f(\mathbf{x_{t-1}}, \hat{v}_t, \hat{\psi}_t) = \mathbf{x_{t-1}} + \left( \hat{v}_t d_t \cos(x_{t-1,3} + \hat{\psi}_t) \, , \hat{v}_t d_t \sin(x_{t-1,3} + \hat{\psi}_t) \, , \hat{v}_t d_t \frac{\sin(\hat{\psi}_t)}{B} \right)^T .$$

The mean (over 50 Monte Carlo runs) map and path estimates at the end of the loop ($T = 1626$) are represented in the first Figure. In this case, the association process and the total number of landmarks are assumed to be known and the proposal distribution is the prior kernel. The second Figure illustrates the variance of the estimation of the robot pose at different times (for each time, the first Figure represents the marginal SLAM and the second one our algorithm).



a) Mean map and path estimates.




b) Error estimate on the robot x-coordinate

## Online EM application to the SLAM problem

- **Linearization step** The normalized complete data log-likelihood at time $t$ is equal to

$$-\sum_{i=1}^{q} \frac{1}{2t} \sum_{s=1}^{t} \mathbf{1}_{i \in \mathcal{A}_s} \left[\mathbf{y_{s,i}} - h(\mathbf{X_s}, \theta_{.,i})\right]^T \times \sigma_{I_{s,i}}^{-2} R^{-1} \left[\mathbf{y_{s,i}} - h(\mathbf{X_s}, \theta_{.,i})\right] ,$$

  up to an additive term that does not depend upon $\theta$. Therefore, it does not belong to the exponential family. For any $i \in \mathcal{A}_s$, $h(\mathbf{x_s}, \theta_{.,i})$ is replaced by

$$h(\mathbf{x_s}, \widehat{\theta}_{.,i}) + \nabla_\theta h(\mathbf{x_s}, \widehat{\theta}_{.,i}) \, (\theta_{.,i} - \widehat{\theta}_{.,i}).$$

- **Online E-step** The linearization step leads to an intermediate quantity of the EM of the form

$$Q_t(\theta, \hat{\theta}) \stackrel{\text{def}}{=} \sum_{i=1}^{q} \langle \mathbb{E}_{\hat{\theta}} \left[\mathsf{S}_{t,i}(\mathbf{Z_{1:t}}, \mathbf{y_{1:t}}) | \mathbf{y_{1:t}}\right], \Xi_i(\theta_{.,i}) \rangle ,$$

with $\mathsf{S}_{t,i}(\mathbf{Z_{1:t}}, \mathbf{y_{1:t}}) = \frac{1}{t} \sum_{s=1}^{t} 1_{i \in \mathcal{A}_s} \mathsf{S}_i(\mathbf{Z_s}, \mathbf{y_{s,i}})$ . We have

$$\mathbb{E}_{\theta'} \left[\mathsf{S}_{t,i}(\mathbf{Z_{1:t}}, \mathbf{y_{1:t}}) | \mathbf{y_{1:t}}\right] = \mathbb{E}_{\theta'} \left[\mathcal{S}^i_{t,\theta'}(\mathbf{Z_t}) | \mathbf{y_{1:t}}\right] ,$$

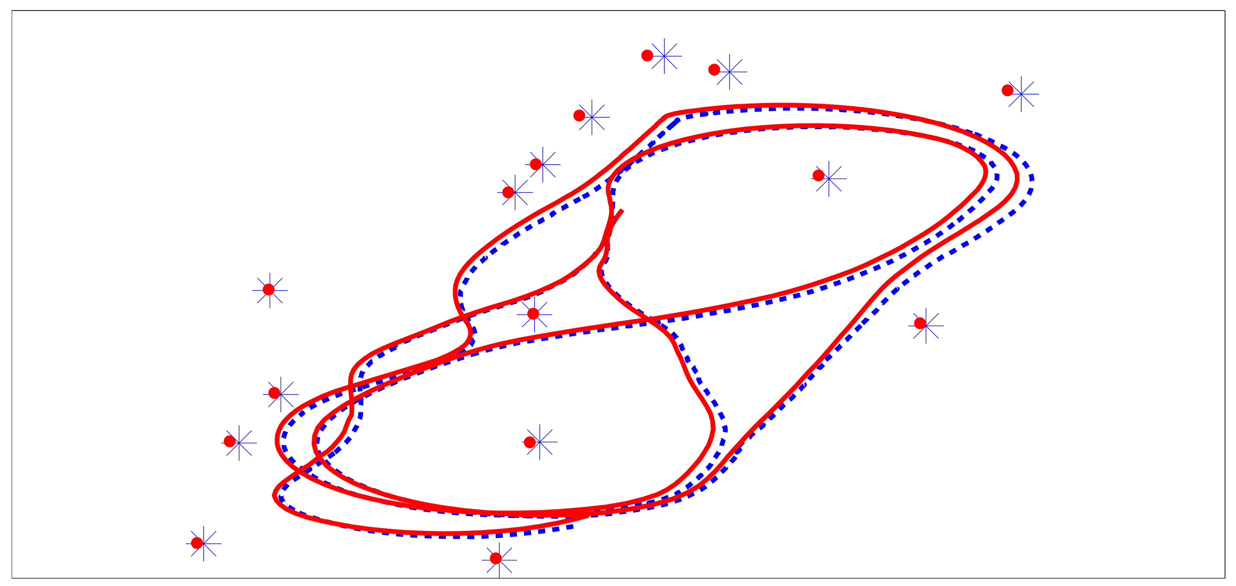
where,

$$\mathcal{S}^i_{t,\theta'}(\mathbf{Z_t}) = \frac{1}{t} 1_{i \in \mathcal{A}_t} \mathsf{S}_i(\mathbf{Z_t}, \mathbf{y_{t,i}}) + \left(1 - \frac{1}{t}\right) \mathbb{E}_{\theta'} \left[\mathcal{S}^i_{t-1,\theta'}(\mathbf{Z_{t-1}}) | \mathbf{Z_t}, \mathbf{y_{1:t-1}}\right] .$$

The E-step necessitates the filtering distribution, the conditional distribution of $\mathbf{Z_{t-1}}$ given $(\mathbf{Z_t}, \mathbf{y_{1:t-1}})$ and a recursive computation of $\mathcal{S}^i_{t,\theta'}$. These distributions are replaced by particle-type approximations and $\mathcal{S}^i_{t,\theta'}(x)$ is updated using a stochastic approximation step.

## Car park data set

We illustrate the performance of the algorithm with real data (see `http://www.cas.kth.se/SLAM` for the data set). The association process is not assumed to be known. The algorithm is initialized with an empty map and each time a new observation is available, its likelihood with respect to each existing landmark is computed. Then, the observation is associated to the landmark giving the largest likelihood or a new landmark is created if all likelihoods are smaller than a given threshold. The last Figure represents the estimated path and map (stars and dotted line) and the true path and map (bold line and dots) at the end of one run.



c) Map and path estimates at the end of the run ($T = 5565$)

## References

[1] O. Cappé and E. Moulines. On-line Expectation Maximization algorithm for latent data models. *J. Roy. Statist. Soc. Ser. B*, 71(3):593–613, 2009.

[2] P. Del Moral, A. Doucet, and S.S. Singh. Forward Smoothing using Sequential Monte Carlo. arXiv:1012.5390, 2010.

[3] S. Le Corff, G. Fort, and E. Moulines. Online Expectation Maximization algorithm to solve the SLAM problem. In *IEEE Workshop Statist. Signal Process*, 2011.

[4] R. Martinez-Cantin. Active map learning for robots: insights into statistical consistency. PhD Thesis, 2008.