Block Online Expectation Maximization for the SLAM

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Introduction

The landmark-based Simultaneous Localization And Mapping (SLAM) problem is written as a problem of inference in a **Hidden Markov Model** (HMM). We consider the case when approximation of the SLAM model by a Linear Gaussian model is not suitable so that Kalman-based solutions (see e.g. [7]) do not apply. We are thus faced with **online** inference in HMM when the (extended) Kalman filter has a very poor behavior.

We propose a solution based on **Expectation Maximization** (EM) type algorithms: we derive the Block Online EM algorithm when the E-step is explicit, and the Particle Block Online EM algorithm otherwise. These algorithms are **streaming** procedures: data are processed only once and need not to be stored.

Consistency of these algorithms is addressed in [4, 5]: the limiting values of the Block Online EM sequences are the stationary points of the limiting normalized log-likelihood of the observations $\lim_{T\to\infty} T^{-1} \log p(y_{1:T};\theta)$ (see [2] for a similar result in the i.i.d.case)

Untractability of EM for online inference in HMM

- Markovian dynamic for the hidden state: $\mathcal{L}(X_t|X_{t-1}) = m_\theta(X_{t-1}, X_t)$.
- Observations governed by the hidden state: $\mathcal{L}(Y_t|X_t) = g_\theta(X_t, Y_t)$

▶ Assumption (exponential model):

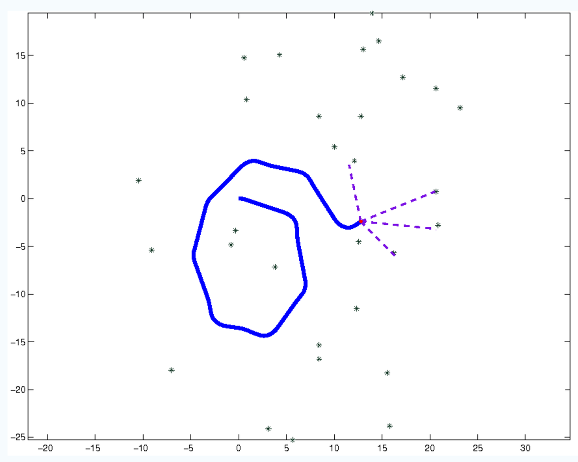
$$\log(m_\theta(x,x')\, g_\theta(x',y)) = \phi(\theta) + \langle S(x,x',y); \psi(\theta)\rangle$$

▶ EM algorithm based on streaming data

- E-step: compute ths statistic $\mathcal{S}_T^{\mathrm{stEM}}(\theta_{T-1}) = \frac{1}{T}\sum_{t=1}^{T} \mathbb{E}_{\theta_{T-1}}\left[S(X_{t-1},X_t,Y_t)|Y_{1:T}\right]$
- M-step: update the parameter $\theta_T = \arg\max_\theta \quad \phi(\theta) + \left\langle \mathcal{S}_T^{\mathrm{stEM}}(\theta_{T-1}); \psi(\theta)\right\rangle$

Unfortunately, *(i)* each iteration necessitates to process all the (past) data; *(ii)* for general HMM, the E-step is not explicit.

Example: SLAM in 2-D



The robot evolves in an unknown environment.

Observation: at each time step, the robot observes the landmarks in a neighborhood.

Mapping: The robot has to find the location of the landmarks.

Localization: The pose of the robot is unknown, and measurements depend on its pose.

Classical model for SLAM: HMM with a hidden state collecting both the map and the pose. But, usual methods are unstable due to the static map.

↪ New model: parameterized HMM. The pose of the robot is the *hidden state* with Markovian dynamic, and this state governs the observations. The transition of the hidden state, and/or the conditional distribution of the observations given the hidden state are *parameterized* by a vector collecting the location of the landmarks.

Block Online EM algorithms for data streams or large data sets

- Choose increasing times: $T_1, T_2, \cdots, T_n, \cdots$ at which the parameter will be updated

▶ Block Online EM algorithm

- E-step: Between time $T_n + 1$ and T_{n+1}, compute ths statistic $\mathcal{S}_{(n)}^{\mathrm{BOEM}}(\theta_{(n)}) \stackrel{\mathrm{def}}{=} \frac{1}{T_{n+1}-T_n}\sum_{t=T_n+1}^{T_{n+1}} \mathbb{E}_{\theta_{(n)}}\left[S(X_{t-1},X_t,Y_t)|Y_{T_n+1:T_{n+1}}\right]$.
- M-step: update the parameter $\theta_{(n+1)} = \arg\max_\theta \quad \phi(\theta) + \left\langle \mathcal{S}_{(n)}^{\mathrm{BOEM}}(\theta_{(n)}); \psi(\theta)\right\rangle$

▶ Particle Block Online EM algorithm:

When the conditional expectation is not explicit, replace it by a *Particle approximation* - Sequential Monte Carlo algorithms for **online** computation of this approximation are proposed in [1, 3].

▶ Averaged (Particle)-BOEM algorithms:

The variability of $\{\theta_{(n)}\}_n$ is reduced when $\mathcal{S}_{(n)}^{\mathrm{BOEM}}(\theta_{(n)})$ is replaced with a weighted linear combination of $\{\mathcal{S}_{(j)}^{\mathrm{BOEM}}(\theta_{(j)})\}_{j\le n}$

Refs.

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Particle BOEM for the SLAM

▶ HMM for the SLAM

Hidden states:

$$X_t = X_{t-1} + \begin{pmatrix} \hat{v}_t\ d_t\ \cos(X_{t-1,3} + \hat{\psi}_t) \\ \hat{v}_t\ d_t\ \sin(X_{t-1,3} + \hat{\psi}_t) \\ \hat{v}_t\ d_t\ B^{-1}\ \sin(\hat{\psi}_t) \end{pmatrix}$$

with controls $(\hat{v}_t, \hat{\psi}_t)_t$ i.i.d.$\mathcal{N}_2(0,Q)$; Q is known.

Observations:

At time t, $(Y_{t,i})_{i\in\mathcal{A}_t}$

$$Y_{t,i} = h(X_t, \theta_{\cdot,i}) + \delta_{t,i}$$

with $\{\delta_{t,i}\}_{t,i}$ i.i.d. $\mathcal{N}_2(0,R)$. R is known.

Exponential model:

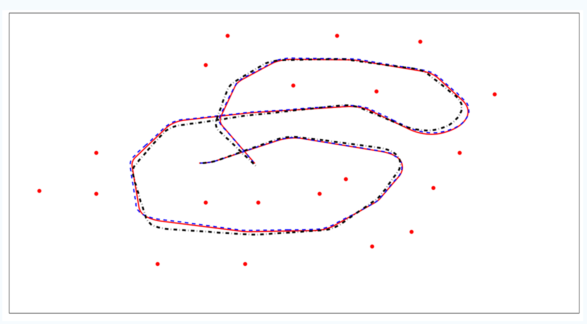
$$h(x,\tau) = \begin{pmatrix} \sqrt{(\tau_1 - x_1)^2 + (\tau_2 - x_2)^2} \\ \arctan\frac{\tau_2 - x_2}{\tau_1 - x_1} - x_3 \end{pmatrix}$$

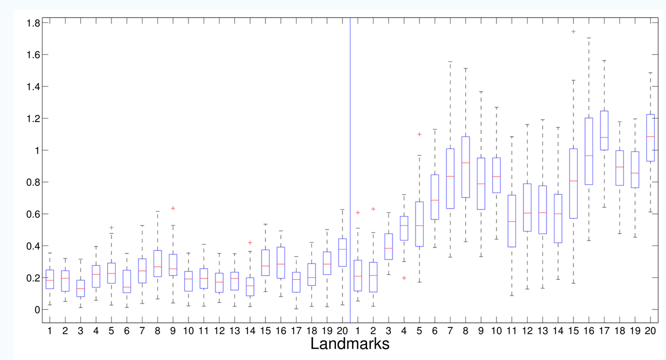
yields the approximation

$$\log g_\theta(x, y_{\cdot,i}) \approx \left[y_{\cdot,i} - \hat{h}(x,\theta_{\cdot,i})\right]^T R^{-1}\left[y_{\cdot,i} - \hat{h}(x,\theta_{\cdot,i})\right]$$

where $\hat{h}(x,\cdot)$ is an approximation of $h(x,\cdot)$, by a local 1st order Taylor expansion.

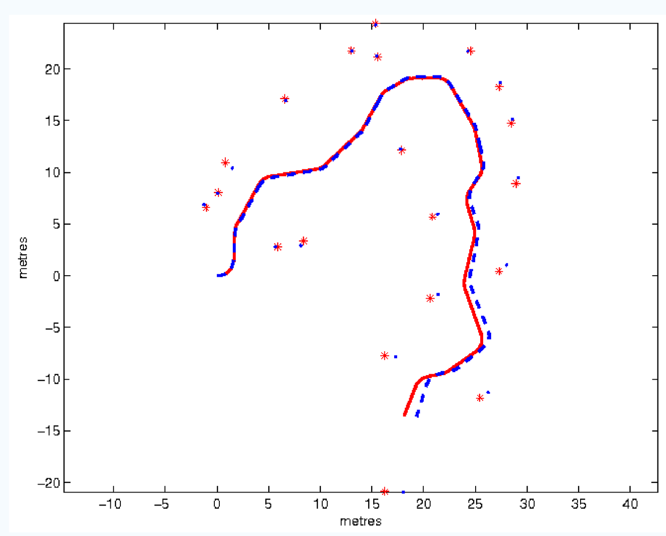
▶ We compare P-BOEM to Marginal-SLAM of [6]; and to EKF (see e.g. [7]).

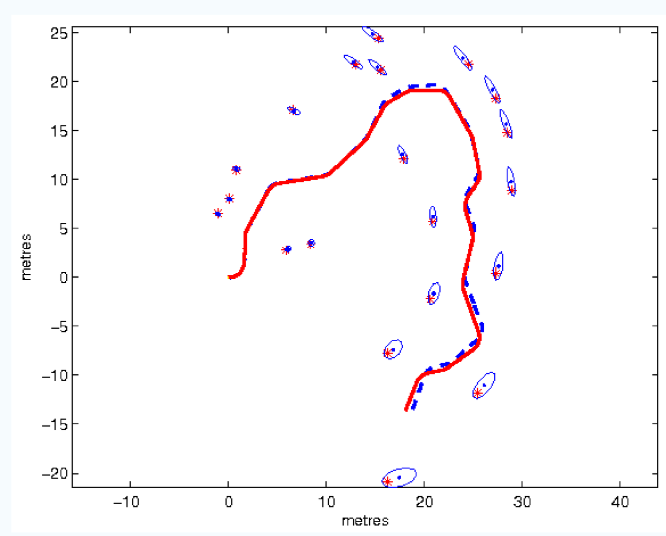




[left] True path (bold red) and estimated path by P-BOEM (dashed blue) and MarginalSLAM (dotted black).

[right] Mean error over 100 indep. run, when estimating each of the 20 landmarks by P-BOEM(left) and Marginal-SLAM (right). After $T = 1800$ obs.





[left] Particle BOEM. [right] EKF