Assimilation of Remote Sensing Data for River Flows

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Abstract. *We address the problem of parameters identification and data assimilation for river flows modeled by the 2D St-Venant equations. In practice, available observations are very sparse especially during flood events (very few measurements of elevation at gauging stations in the main channel). We assume we have in addition either surface trajectories extracted from video images (lagrangian data) or space distributed water levels extracted from one satellite image. Then we identify parameters such as the inflow discharge or the topography and/or the initial state (depending on the configuration and the observations available). Numerical twin experiments demonstrate the efficiency of the present method for toy test cases.*

1   Introduction

A major difficulty in numerical simulation of river hydraulics is to calibrate the models. Variational data assimilation (VDA) combines, in an optimal sense, model and observations allowing to identify some parameters values. However, in river hydraulics, observation data are available only in very small quantities. Water level measurements can be available at gauging stations, but it is the main channel only and they are very sparse in space. Velocity measurements are even rarer and uncertain, since they require complex human interventions. Consequently, in practice these observations are usually not sufficient to take full advantage of data assimilation. This lack of data becomes even more problematic in case of floods. Thus, remote sensed data such as videos and spatial images offer a large potential which is not exploited quantitatively yet. We address the assimilation of the following two types of data (in addition to classical ones) : 1) trajectories of particles flowing at the surface which could be extracted from video images; 2) spatial distributed water level which could be extracted from a satellite image of the flood plain.
In case 1), we extend the method of VDA to extra lagrangian data. The trajectories of particles convected by the flow bring information on the surface velocity using an extra transport model. In case 2), we define an extra cost function term.
Numerical twin experiments done for toy test cases show that one improves the identification of model parameters.

The forward model and VDA process The forward model relies on the 2D St-Venant model (shallow water equations SWE) (h is the water elevation, $\mathbf{q} = h\mathbf{u}$ the discharge, $\mathbf{u}$ the depth-averaged velocity):

$$\begin{cases} \partial_t h + \text{div}(\mathbf{q}) = 0 & \text{in } \Omega \times]0,T] \\ \partial_t \mathbf{q} + \text{div}(\frac{1}{h}\mathbf{q} \otimes \mathbf{q}) + \frac{1}{2}g\nabla h^2 + gh\nabla z_b + g\frac{n^2\|\mathbf{q}\|}{h^{7/3}}\mathbf{q} = 0 & \text{in } \Omega \times]0,T] \end{cases} \quad (1)$$

with initial conditions $(h_0, \mathbf{q}_0)$ given, g the gravity, z_b the bed elevation, n the Manning roughness coefficient. Boundary conditions are: at inflow, the discharge $\bar{q}$ is prescribed; at outflow, either the water elevation $\bar{z}_s$ is prescribed or incoming characteristics are prescribed; and walls conditions. We refer to [3] for more details.

Given the control vector $\mathbf{c} = (h_0, \mathbf{q}_0, n, z_b, \bar{q}, \bar{z}_s)$, the state variable $(h, \mathbf{q})$ is determined by solving the forward model.

The full VDA process based on the optimal control method [7], is implemented into our software DassFlow, [3]. The forward code is a HLLC Riemann solver, [1]. The cost functions are minimized using a Quasi-Newton algorithm, which implies the computation of its gradient using an adjoint model. The latter is created with the help of the automatic differentiation tool Tapenade, [2].

2 Assimilation of lagrangian data

Lagrangian DA consists in using observations described by lagrangian coordinates in the VDA process. Here, we consider observations of particles transported by the flow (e.g. extracted from video images). The link between the lagrangian data made of N particle trajectories denoted by $X_i(t)$ and the classical eulerian variables of the shallow water model is made by the following equations, see [4]:

$$\begin{cases} \frac{d}{dt}X_i(t) = \gamma\mathbf{u}\Big(X_i(t),t\Big) & \forall t \in]t_i^0, t_i^f[\\ X_i(t_i^0) = x_i^0 , \end{cases} \quad \text{for } i = 1, \ldots, N \quad (2)$$

where t_i^0 and t_i^f are the time when the particle enter and leave the observation domain, γ is a multiplicative constant. We consider two kinds of observations (classical eulerian observations $h^{obs}(t)$ and trajectories of particles transported by the flow $X_i^{obs}(t)$). Then, we build the following composite cost function:

$$j(\mathbf{c}) = \frac{1}{2}\int_0^T \big\| Ch(t) - h^{obs}(t) \big\|^2 dt + \frac{\alpha_t}{2}\sum_{i=1}^N \int_{t_i^0}^{t_i^f} \big| X_i(t) - X_i^{obs}(t) \big|^2 dt \quad (3)$$

where α_t is a scaling parameter, C the observation operator (restriction). $j(\mathbf{c})$ is the cost function we minimize with respect to $\mathbf{c}$.

Numerical results. Particle trajectories associated with local water depth measurements are used for the joint identification of local bed elevation z_b and initial conditions $(h^0, \mathbf{u}^0)$, Fig. 1. Twin DA experiments are carried out. Observations are: (1) water depth recorded continuously in time at the abscissae $x_1 = 15\,m$ and $x_2 = 70\,m$, for the whole width of the domain; (2) virtual particles dropped in the flow (640 in total) and transported by a turbulent surface velocity $\mathbf{u}^t = \gamma\mathbf{u} + \mathbf{u}^p$, where $\gamma = 1$ and $\mathbf{u}^p$ is a Gauss-Markov process. Then these observations are

filtered by averaging, see [4].

We identify jointly the reference topography and the reference initial conditions by minimizing (3). (The *a priori* value of the control variable was a constant slope bed and the resulting steady-state flow as initial conditions. The identified topography we obtain is close to the reference one, Fig. 1(b); while the identified initial conditions reproduces well the reference one.

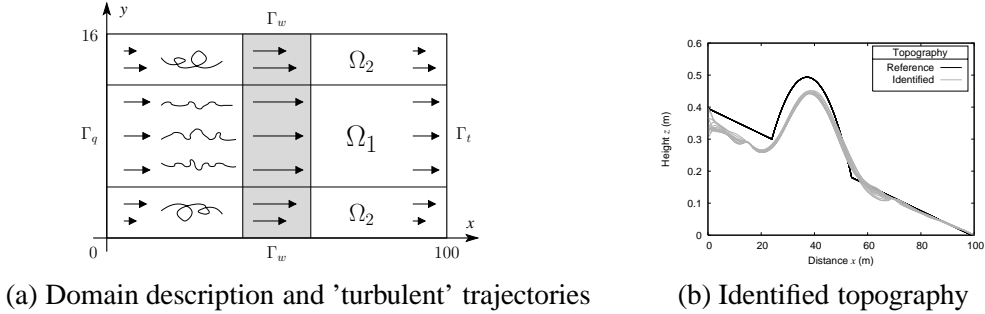(a) Domain description and 'turbulent' trajectories (b) Identified topography

Figure 1: Joint identification of the topography and the initial conditions using water depth measurements and particle trajectories with $\alpha_t = 1 \times 10^{-4}$.

3   Assimilation of spatial distributed water levels

We consider the toy flood event described in Fig 2 and 3. This toy test case contains all important features of the real case studied in [5] (Moselle river). We seek to identify the inflow discharge only (other parameters are given). Available observations are, see Fig. 2 (b): (ObsA) the water level measured at the gauge station (partially in time); (ObsB) the water levels extracted from a virtual image and available in three blocks only. Concerning the cost function, in addition of the classical term related to the h observations (ObsA) and a regularization term ($\|\partial_t c\|^2$), we introduce the following extra term (where information is available):

$$J_{flux}(\mathbf{c}) \; = \; \frac{1}{2} \int_0^T \|\tilde{C}\mathbf{q}(t) - \tilde{\mathbf{q}}^{obs}(t)\|^2 \, dt \tag{4}$$

where q is the computed discharge (net mass flux) and $\tilde{\mathbf{q}}^{obs}$ is a "mix" net mass flux since it is computed using the observed elevation h^{obs} and the computed velocity $\mathbf{u}$, $\tilde{\mathbf{q}}^{obs} = h^{obs}\mathbf{u}$. We refer to [6] for more details.

Numerical results.   Fig. 3 (b) shows that this extra term J_{flux} improves the minimization process since it quantifies a discrepancy related to the second component of the state variable. Also, Fig. 3 (b) shows that if we assimilate both (ObsA) and (ObsB) then one obtains a quite good identification of inflow discharge. At contrary, without the contribution of the "partial image" ((ObsA) only is available), the identification process fails.

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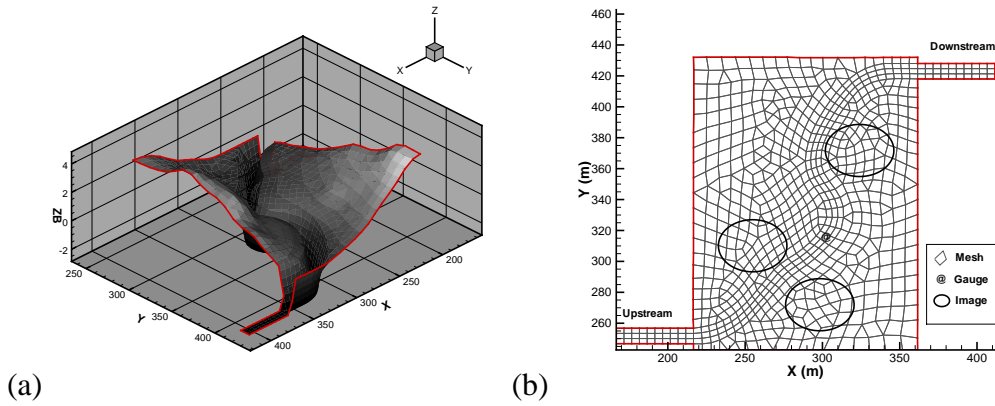
(a)   (b)

Figure 2: Spatial distributed water levels, the toy test case. (a): topography; (b): mesh, gauge station and observed areas at image time.
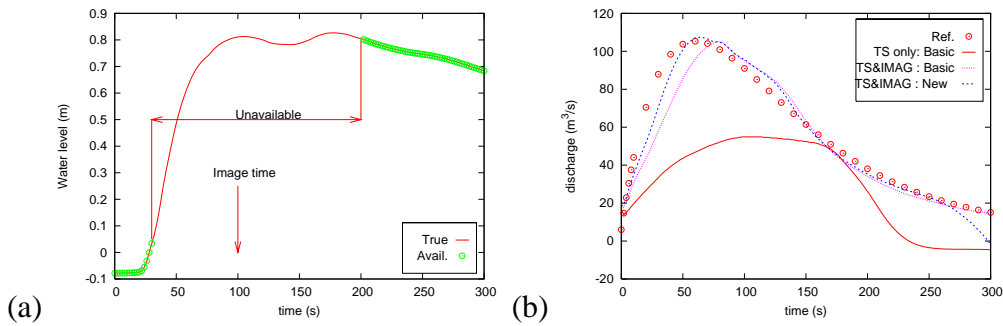

(a)   (b)

Figure 3: (a) Measures in-situ (gauge station) partially available in time and image time. (b) Identified inflow discharge: comparison if the image is available or not; also if considering the extra term J_{flux} or not.

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