DIFFERENTIATION OF REAL FUNCTIONS ALONG RECTANGLES

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Given a family R of rectangles in $\mathbb{R}^n$ of the form $[0,\alpha_1]\times\cdots\times[0,\alpha_n]$, we let $B := \{\tau(I) : I \in R, \tau \text{ translation}\}$ be the associated (translation-invariant) differentiation basis and define a maximal operator M_R by:

$$M_R f(x) := \sup_{I\in B, I\ni x} \frac{1}{|I|}\int_I |f|.$$

Given $X \subseteq L^1(\mathbb{R}^n)$ an Orlicz space, it is often the case that the two following properties are equivalent:

(A) $M_R f(x) < +\infty$ for a.e. $x \in \mathbb{R}^n$;

(B) R differentiates X in the sense that for all $f \in X$, one has:

$$f(x) = \lim_{x\in I\in R, \operatorname{diam} I\to 0} \frac{1}{|I|}\int_I f,$$

for a.e. $x \in \mathbb{R}^n$.

In this talk we shall discuss some geometrical properties on R that guarantee or not the validity of properties (A) & (B) above for some classical Orlicz spaces X. We shall particularly focus on the case $n = 2$, survey the classical results obtained in this case, and see how things change in the plane when rectangles from R are allowed to rotate around their lower left vertex, with an angle belonging to some small set. If time allows us to do so, we shall also discuss recent results obtained jointly with E. D'Aniello in the n-dimensional case.