FROM UNBALANCED OPTIMAL TRANSPORT TO THE CAMASSA-HOLM EQUATION

FRANÇOIS-XAVIER VIALARD

In the first part of the talk, we present a natural extension of the Wasserstein L2 metric on the space of probablility measures to the space of positive Radon measures that do not have the same total mass. This generalization can be seen as a Riemannian analog of the flat norm on the space of measures. We introduce it as the infimal convolution of the Wasserstein and the Fisher-Rao metric using a dynamic formulation. We then show its equivalent static formulation efficient for computations. In the second part, we present, via a generalization of Otto's Riemannian submersion the link between this new metric on the space of probabilities and the Camassa-Holm equation. The Camassa-Holm equation is a geodesic equation for a right-invariant metric on the group of diffeomorphisms. Our point of view gives an isometric embedding of the group in a Hilbert manifold which has interesting analytical consequences.