Rational maps with Cluster Cycles and the Mating of Polynomials

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11th June 2011
Workshop on the Matings of Polynomials
Institut de Mathématiques de Toulouse

Outline

Definitions.

Let $f : \widehat{\mathbb{C}} \to \widehat{\mathbb{C}}$ be a (bicritical) rational map.

- The Julia set $J(f)$ is the closure of the set of repelling periodic points of f.
- The Fatou set $F(f)$ is $\widehat{\mathbb{C}} \setminus J(f)$.

If f is a polynomial

- The filled Julia set is $K(f) = \{z \in \widehat{\mathbb{C}} \mid f^{\circ n}(z) \not\to \infty\}$, so that $J(f) = \partial K(f)$

In this talk, we will generally assume that f has a (finite) superattracting periodic cycle of period $p > 1$.

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Definitions

Suppose $f_c(z) = z^d + c$. Recall the definition of the Carathéodory loop, γ. Then we see

- The points $\beta_k = \gamma(k/(d-1))$, $k = 0, 1, \ldots, d-2$ are fixed points on $J(f)$.
- If $\alpha \in J(f)$ is the other fixed point and α is the landing point of the ray of angle θ, then it is also the landing point of the rays of angle $d\theta, d^2\theta, \ldots$
- Indeed, if $z = \gamma(\theta)$, then $f(z) = \gamma(d\theta)$.

Definition

A multicurve $\Gamma = \{\gamma_1, \ldots, \gamma_n\}$ of F is called a Levy cycle if for $i = 1, 2, \ldots, n$, the curve γ_{i-1} is homotopic (rel P_F) to a component γ'_{i-1} of $F^{-1}(\gamma_i)$ and the map $F : \gamma'_i \to \gamma_i$ is a homeomorphism.

Thurston's Theorem

Two branched covers F and G are said to be Thurston equivalent if $\exists$ orientation preserving homeomorphisms $\phi_0, \phi_1 : S^2 \to S^2$:

- $\phi_0|_{P_F} = \phi_1|_{P_F}$
- $\phi_1 \circ F = G \circ \phi_0$
- ϕ_0 and ϕ_1 are isotopic through ϕ_t, $t \in [0,1]$, $\phi_0|_{P_F} = \phi_t|_{P_F} = \phi_1|_{P_F}$ for $t \in [0,1]$.

Theorem (Thurston)

Let $F : S^2 \to S^2$ be a postcritically finite branched cover with hyperbolic orbifold. Then F is equivalent to a rational map if and only if F has no Thurston obstructions. This rational map is unique up to Möbius transformation.

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In general it is difficult to find Thurston obstructions. Levy cycles simplify the search.

- F has a Levy cycle $\Rightarrow$ F has a Thurston obstruction.
- In the bicritical case: F has a Thurston obstruction $\Rightarrow$ F has a Levy cycle.

Theorem (Rees, Shishikura, Tan L.)

In the bicritical case, if $[\alpha_1] \neq [\alpha_2]$, $K_1 \perp\!\!\!\perp K_2$ is homeomorphic to S^2 and we can give this sphere a unique conformal structure to make $f_1 \perp\!\!\!\perp f_2$ a holomorphic degree d rational map.

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Simplifying Thurston's criterion

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Definition

Let F be a bicritical rational map such that the two critical points belong to the attracting basins of two disjoint (super)attracting periodic orbits of the same period.

Clustering is the condition that the critical orbit Fatou components group together to form a periodic cycle...

- The dynamics on each Fatou component can be conjugated using Böttcher's theorem.
  - Internal rays
- The 0 internal ray is fixed under the first return map.
- If the 0 internal rays meet at a point c, and this point is periodic, we say c is a cluster point for F.

e.g. Rabbit $\perp\!\!\!\perp$ Airplane.

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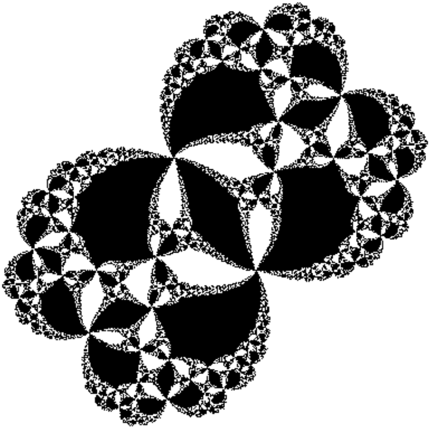
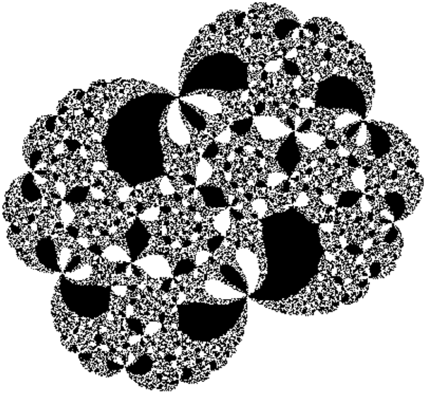


The Julia set for Rabbit $\perp\!\!\!\perp$ Airplane
(and Airplane $\perp\!\!\!\perp$ Rabbit!).

Another example



The Julia set for a map with a period two cluster cycle.

Combinatorial data

Restrict attention to the period one and two cases.
We can describe the cluster in simple combinatorial terms.

1. The period of the critical cycles n.
2. The combinatorial rotation number ρ.
3. The critical displacement δ.

The combinatorial data of the cluster will be the pair (ρ, δ).

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Clusters and Thurston Equivalence

Theorem (S., 2010)

If F and G are bicritical rational maps of the same degree and

- *F and G have fixed cluster points and are of degree d*
- *F and G are quadratic with a period 2 cluster cycle*

then F and G have the same combinatorial data if and only if they are Thurston equivalent.

The above is false in the case where F and G have degree $d \geq 3$ and a period two cluster cycle.

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Proof Outline

In both cases we use the following method. let X_F and X_G be the union of the stars of the clusters of F and G.

1. There exists $\phi \colon X_F \to X_G$ which conjugates the dynamics.
2. Extend ϕ to a map $\Phi \colon \widehat{\mathbb{C}} \to \widehat{\mathbb{C}}$.
3. Construct a new map $\widehat{\Phi}$ such that $\Phi \circ F = G \circ \widehat{\Phi}$.
4. Modify $\widehat{\Phi}$ so that it is equal to ϕ on X_F.
5. Modify this new $\widehat{\Phi}$ so that it is isotopic to Φ rel X_F.

We get a commutative diagram. . .

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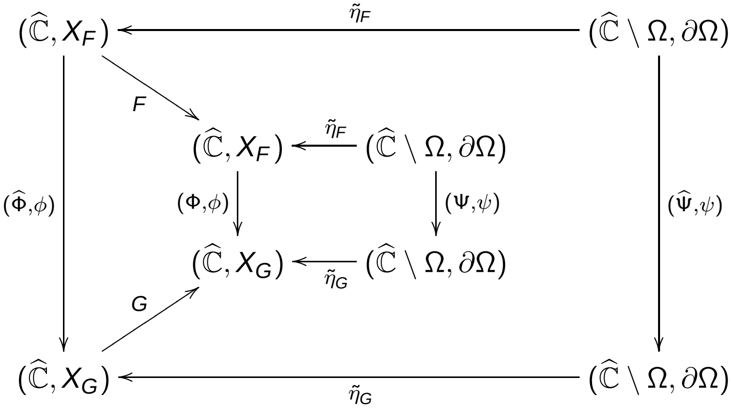

Ω is either $\mathbb{D}$ (fixed case) or an annulus (period 2 case).

$$(\widehat{\mathbb{C}}, X_F) \qquad\qquad\qquad\qquad (\widehat{\mathbb{C}} \setminus \Omega, \partial\Omega)$$

$$\begin{array}{ccc} (\widehat{\mathbb{C}}, X_F) & & (\widehat{\mathbb{C}} \setminus \Omega, \partial\Omega) \\ \phi \downarrow & & \\ (\widehat{\mathbb{C}}, X_G) & & (\widehat{\mathbb{C}} \setminus \Omega, \partial\Omega) \end{array}$$

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ϕ is a conjugacy.

Constructing the diagram

$$(\widehat{\mathbb{C}}, X_F) \qquad\qquad (\widehat{\mathbb{C}} \setminus \Omega, \partial\Omega)$$

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$\tilde{\eta}_F$ and $\tilde{\eta}_G$ are Riemann maps. ψ is induced by ϕ.

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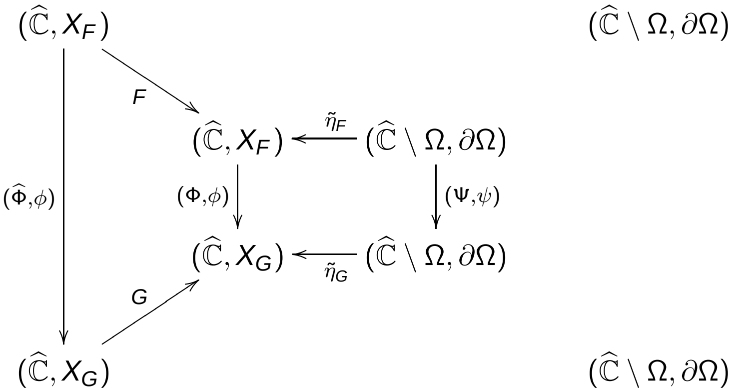
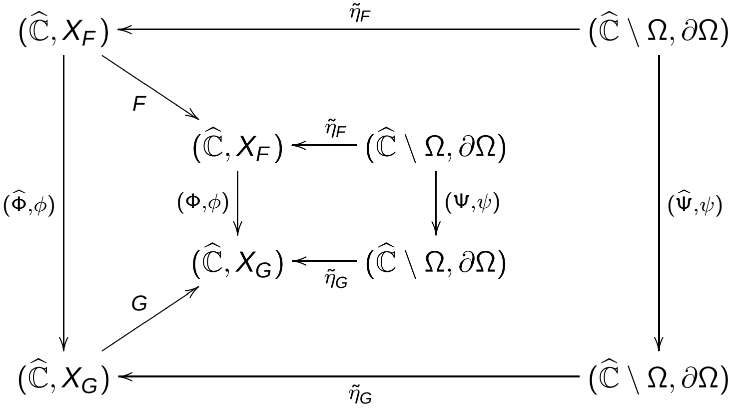
ψ extends to the homeomorphism Ψ which induces the homeomorphism Φ.

Constructing the diagram



Construct $\widehat{\Phi}^{!}$ so the diagram commutes.

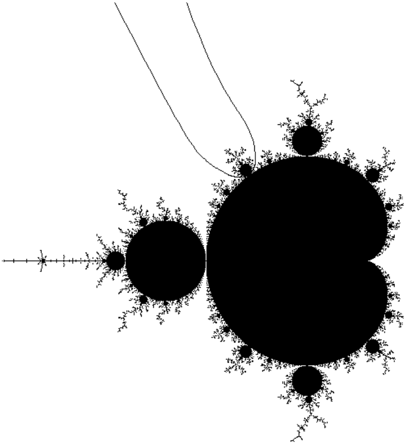
Constructing the diagram



Finally, get the induced map $\widehat{\Psi}$ and check if it is isotopic to Ψ.

Period 1 Results

A rabbit is any map with a “star-shaped” Hubbard tree. They belong to hyperbolic components which bifurcate from the (unique) period one component in the Multibrot set.



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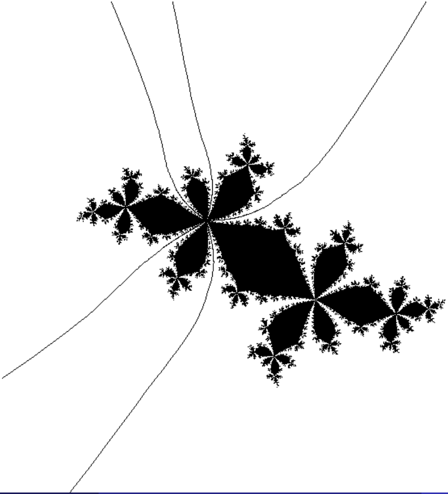

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Lemma

If $F = f_1 \perp\!\!\!\perp f_2$ has a fixed cluster point, then precisely one of the f_i is a rabbit.

Lemma

All combinatorial data can be realised (in precisely $2d - 2$ ways), save for the case with $\delta = 1$ or $\delta = 2n - 1$.

The rotation number is fixed by the rotation number of the α-fixed point for the rabbit. The critical displacement is determined by the choice of the complementary map.

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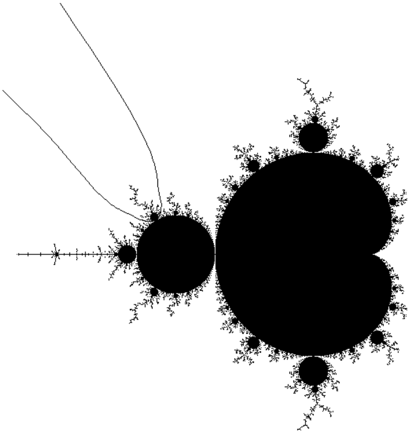
Period 2 (quadratic) Results

A bi-rabbit is a map bifurcating off the period 2 component.



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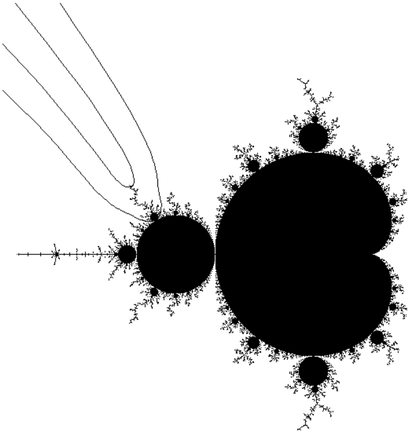

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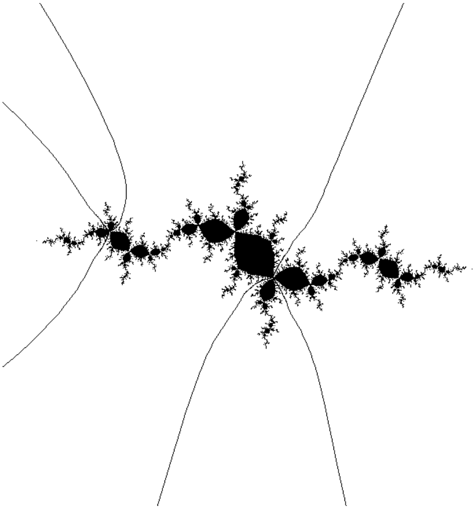

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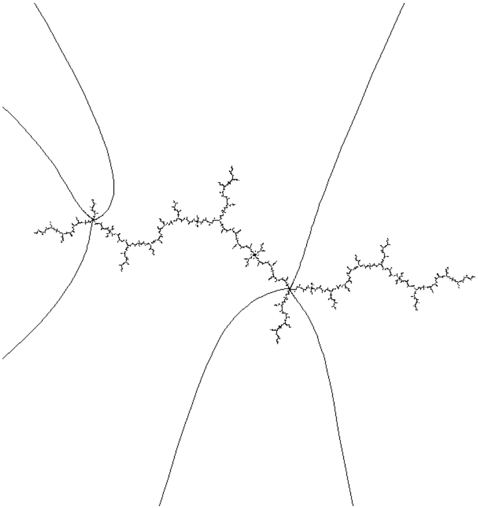

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Theorem (S., 2009)

If $F = f_1 \amalg f_2$ has a period two cluster cycle, one of the f_i is either a bi-rabbit or a secondary map which lies in the limb of the bi-rabbit.

Lemma (S., 2009)

All combinatorial data can be realised (in at least two ways).

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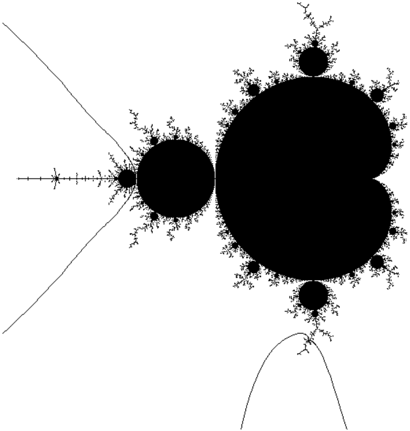
Example

The mating of these two components...



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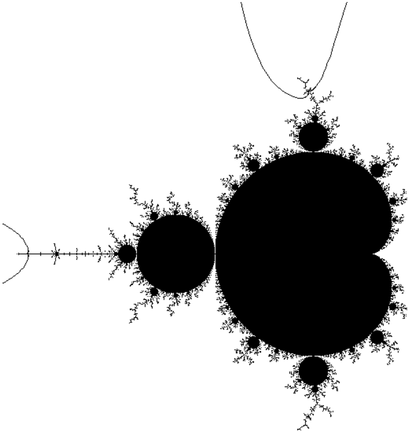

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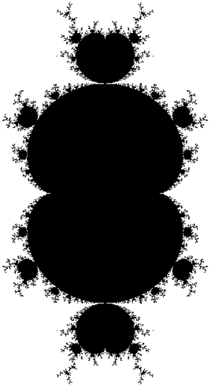
A counterexample

Consider the degree 3 multibrot set.



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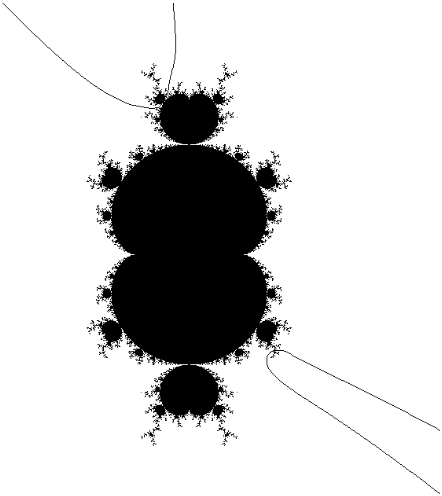
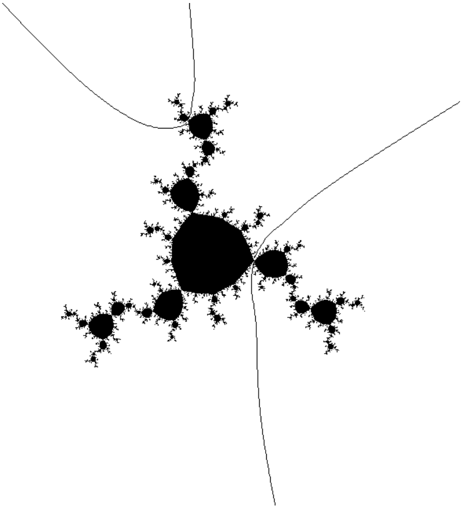
The mating of these two components is a rational map with combinatorial data $(\rho, \delta) = (1/2, 3)$.



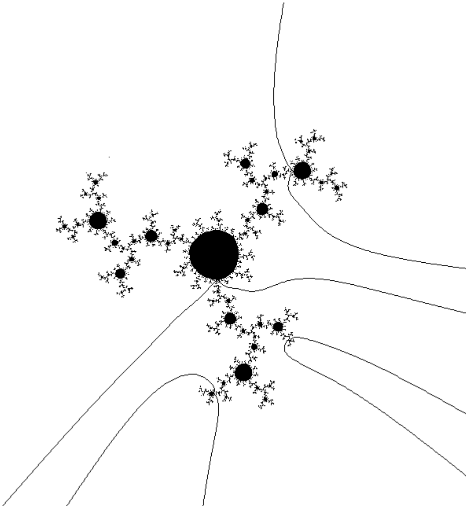
A counterexample

This is the mating of a bi-rabbit. . .



A counterexample

. . . with a complementary map.



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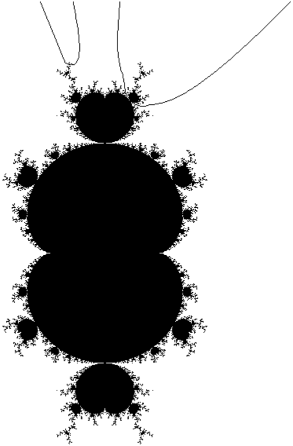
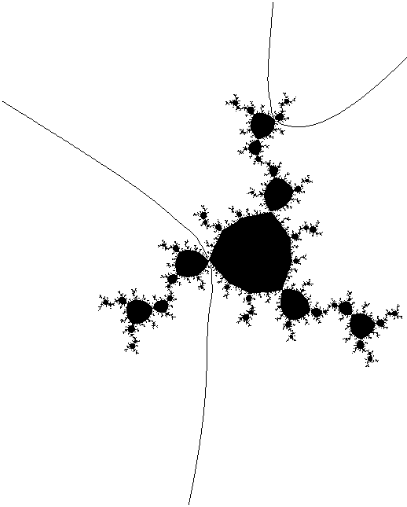


A counterexample

The first map is a bi-rabbit. . .



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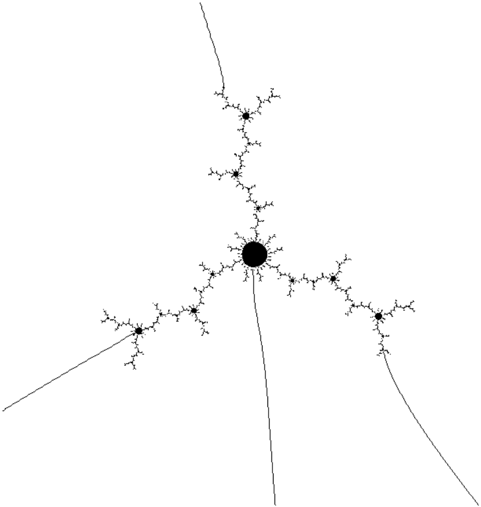
. . . the other map lies beyond the same period two component



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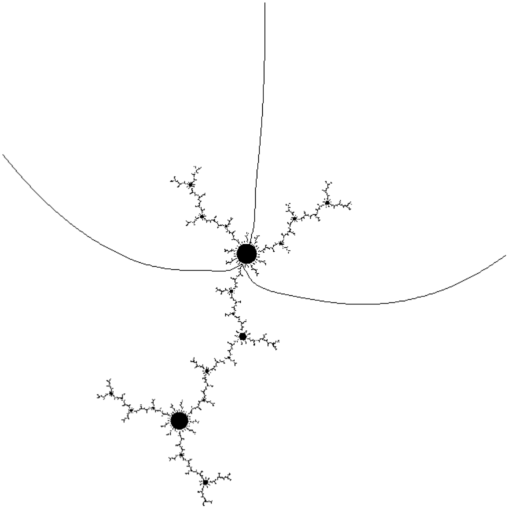
A closer look at the critical value component for this map.



Observations

These two examples allow us to observe

- both maps have the same *intrinsic* combinatorial data
- the two rational maps formed by the matings are different
- the first mating is analogous to the degree 2 case, the second is a different kind of mating... in higher degrees, we have the existence of *non-principal* root points.

So in this case the combinatorial data is not enough to classify the rational maps in the sense of Thurston.
There is a difference between the degree 2 and the bicritical cases.

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- both maps have the same *intrinsic* combinatorial data
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- the first mating is analogous to the degree 2 case, the second is a different kind of mating. . . in higher degrees, we have the existence of *non-principal* root points.

So in this case the combinatorial data is not enough to classify the rational maps in the sense of Thurston.
There is a difference between the degree 2 and the bicritical cases.

Summary

- In simple cases, the combinatorial data of a cluster completely defines a rational map.
  - Period 3?
- Period 1 and period 2 cases are very similar, but with an increased level of complexity for the period 2 case.
  - "Non-trivial" shared matings
  - Different combinatorial data
  - Simple Thurston classification only works in the quadratic case for period 2
- Combinatorics of the matings (not discussed in the talk)

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  - Positions of fixed points
  - Possibly a classification of the "2 period 2" case will shed light on the problem
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  - More "secondary maps"
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