

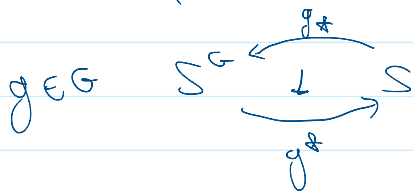
last time : S , $\mathcal{C} = S^G$, $\text{Set} \xrightarrow{\text{}} S \xrightarrow{(\mathbb{1}_G)^*} S^G = \mathcal{C}$

\mathcal{O} operad in \mathcal{C}

$\partial D^d \hookrightarrow D^d$ cofibration in Set

its geom realization is homeomorphic to $\partial D^d \hookrightarrow D^d$

$\partial D^d = \partial |D^d| \rightarrow |D^d| = D^d$



$X \mapsto X(g)$

• Cellular algebra

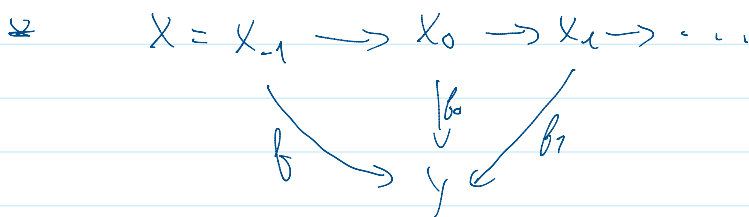
"iterated attachment of cells"

Definition: $X \in \text{Alg}_{\mathcal{O}}(\mathcal{C})$ is a cellular algebra if $i \rightarrow X$

i is a cellular map

Def: $f: X \rightarrow Y$ in $\text{Alg}_{\mathcal{O}}(\mathcal{C})$ is a cellular map if it

is a transfinite composition of cell attachments. This means



Indexed over cardinal κ

* $\text{colim}_{i \in \mathbb{N}} f_i$ is an isomorphism

* $i \in \mathbb{N}$ succession

$$f^{\circ} \left(\coprod_{d \in \mathbb{N}} \partial \mathcal{D}^{d,d} \right)$$

$$f^{\circ} \left(\coprod_{d \in \mathbb{N}} \partial \mathcal{D}^{d,d} \right) \xrightarrow{\coprod h_d} X_{i-1}$$

$$\begin{array}{ccc} \downarrow & \text{pencil} & \downarrow \\ f^{\circ} \left(\coprod_{d \in \mathbb{N}} \partial \mathcal{D}^{d,d} \right) & \longrightarrow & X_i \end{array}$$

$$h_d : \partial \mathcal{D}^d \rightarrow \mathcal{U}^{\circ} (X_{i-1}(g_d)) \text{ in } \mathcal{S}$$



$$h_d : f^{\circ}(\partial \mathcal{D}^{d,d}) \rightarrow X_{i-1} \text{ in } \text{Alg}_{\mathcal{O}}(e)$$

* $i \in \mathbb{N}$ limit

$$\text{colim}_{i' < i} f_{i'} = f_i : X_i \rightarrow Y$$

→ We want an object in $\text{Alg}_{\mathcal{O}}(e^{\mathbb{Z}_{\mathbb{N}}}) \neq \text{Alg}_{\mathcal{O}}(e^{\mathbb{Z}_{\mathbb{N}}})$

Def: $\partial \mathcal{D}^d \hookrightarrow \mathcal{D}^d$ in sSet

$$d_* \mathcal{D}^d = \mathcal{D}^d[\mathcal{D}] = (\dots \rightarrow \emptyset \rightarrow \mathcal{D}^d \xrightarrow{d} \mathcal{D}^d \xrightarrow{d} \mathcal{D}^d \rightarrow \dots)$$

↑
↑
↑
↑

$$\partial \mathcal{D}^d[\mathcal{D}] = (\dots \rightarrow \emptyset \rightarrow \partial \mathcal{D}^d \rightarrow \mathcal{D}^d \rightarrow \partial \mathcal{D}^d \rightarrow \dots)$$

objects in $\mathcal{S}\text{Set}^{\mathbb{Z}}$

$$\mathcal{S}\text{Set} \rightarrow \mathcal{S} \rightarrow \mathcal{S}^G = \mathcal{C}$$

We consider that $\mathcal{D}^d[\mathcal{D}], \partial \mathcal{D}^d[\mathcal{D}] \in \mathcal{C}^{\mathbb{Z}}$

$$X \in \mathcal{C}^{\mathbb{Z}} \cong \mathcal{C}^{G \times \mathbb{Z}} \quad X(g, d) \in \mathcal{S}$$

Def: $X \in \text{Alg}_{\mathcal{C}}(\mathcal{C})$ is a CW-algebra if $i \rightarrow X$ is a relative CW-structure

Def: $f: X \rightarrow Y$ in $\text{Alg}_{\mathcal{C}}(\mathcal{C})$ is a (countable) diagram in $\text{Alg}_{\mathcal{C}}(\mathcal{C}^{\mathbb{Z}})$

$$O_k(X) = \text{St}_{k-1} \xrightarrow{b_0} \text{St}_k \xrightarrow{b_1} \text{St}_{k+1} \rightarrow \dots$$

* For each $d \geq 0$, we have \mathbb{Z}_d set $\{g_d \in G \mid d \in \mathbb{Z}_d\}$

and morphisms $e_d: \partial \mathcal{D}_d^d \rightarrow \mathcal{C}^{\mathbb{Z}} \text{St}_{d-1}(g_d, d-1)$ in \mathcal{S}

adjoint to

$$\begin{array}{ccc}
 \mathcal{C}^{\mathbb{Z}}(\partial \mathcal{D}^d[\mathcal{D}]) & \xrightarrow{\text{Uev}} & \text{St}_{d-1} \in \text{Alg}_{\mathcal{C}}(\mathcal{C}^{\mathbb{Z}}) \\
 \downarrow & & \downarrow b_d \\
 \mathcal{C}^{\mathbb{Z}}(\coprod \mathcal{D}^{g_d, d}[\mathcal{D}]) & \longrightarrow & \text{St}_d
 \end{array}$$

$$\mathcal{F}^{\sigma} \left(\coprod_{\alpha \in \mathcal{A}} \mathcal{D}^{\sigma, \alpha} [\mathcal{A}] \right) \longrightarrow \text{Std}$$

$$\text{in } \text{Alg}_{\sigma} (e^{\mathbb{Z}_{\sigma}})$$

$$* \text{Std} = \text{colim}_{\mathcal{I}} \text{Std}_{\mathcal{I}} \in \text{Alg}_{\sigma} (e^{\mathbb{Z}_{\sigma}})$$

$$\begin{array}{ccc} X & \xrightarrow{f} & Y \\ \cong \downarrow & & \downarrow \cong \end{array} \quad \text{in } \text{Alg}_{\sigma} (\mathcal{C})$$

$$\begin{array}{ccc} \text{colim}(\mathcal{F}_{\mathcal{I}}) & \longrightarrow & \text{colim}(\text{Std}) \\ \downarrow & & \uparrow \\ \text{colim}(\text{Std}_{\mathcal{I}_0}) & \longrightarrow & \text{colim}(\text{Std}_{\mathcal{I}_1}) \end{array}$$

Theorem: let $X \rightarrow Y$ (lex map)
 $\text{gr}(\text{Std}) \text{ in } \text{Alg}_{\sigma} (e^{\mathbb{Z}_{\sigma}})$

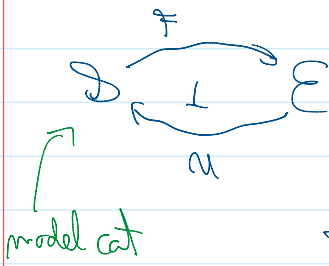
$$\mathcal{O}_{\sigma}(X) \xrightarrow{\quad} \bigvee^{\sigma} \mathcal{F}^{\sigma} \left(\bigvee_{\mathcal{A} \neq \emptyset} \bigvee_{\alpha \in \mathcal{I}_{\mathcal{A}}} \mathcal{D}_{\sigma}^{\sigma, \alpha} (\mathcal{F}_{\sigma}^{\sigma, \alpha}) \right)$$

Coproduct in $\text{Alg}_{\sigma} (e^{\mathbb{Z}_{\sigma}})$

$$\mathcal{F}^{\sigma, \alpha} = \text{colim}_{\text{in } \mathcal{C}} \left(\begin{array}{ccc} \mathcal{D}^{\sigma, \alpha} & \longrightarrow & \mathcal{D}^{\sigma, \alpha} \\ \downarrow & & \\ * & & \end{array} \right) \in \mathcal{C}_{*}$$

• Model categories

• Model categories



Projective model structure

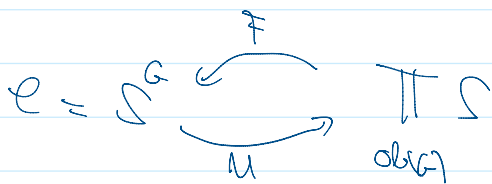
• morphism in \mathcal{E}

→ fib if $U(f) \text{ fib}$

→ w.e. $\ll \ll$ w.e

⇒ (sometimes) \mathcal{E} becomes a model cat

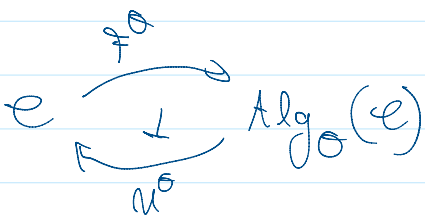
• S (cofibrantly gen) model cat



⇒ \mathcal{C} projective model cat

$$x \longmapsto (X(g))_{g \in G}$$

$$f((X_g)_{g \in G}) = \bigsqcup_{g \in G} \text{hom}_G(g, -) \times X_g$$



⇒ projective model str on $\text{Alg}_O(\mathcal{C})$

Prop: To A , \dots $(+ \dots +)$ \dots $(+ \dots +)$

Prop:

If Θ is cofibrant (the underlying Segm seq is cofibrant)

$\Rightarrow U^{\Theta}$ preserves cofibrations between cofibrant objects.