Report of the ANR project "Mathematical Analysis of Topological Singularities in some physical problems" (MAToS), 2015-2019

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1 Introduction

This is the final report of the ANR project 14-CE25-0009-01 entitled "Mathematical Analysis of Topological Singularities in some physical problems" (MAToS) that was developed by the authors between January 2015-December 2019. The central theme of this project lied in the area of nonlinear analysis (nonlinear partial differential equations and calculus of variations). We focused on the structure and dynamics of topological singularities arising in some variational physical models driven by the Landau-Lifshitz equation (in micromagnetics) and the Gross-Pitaevskii equation (in superconductivity, Bose-Einstein condensation, nonlinear optics). These included vortex singularities, traveling waves and domain walls in magnetic thin films. These structures are observed experimentally and in numerical simulations and play an important role in the dynamics of the corresponding physical systems. We made significant progress in the mathematical analysis of these structures (both at the stationary and dynamical level) that gives more insight into the physical phenomena.

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2 State of art

We tackled new and difficult mathematical problems on the structure and dynamics of topological singularities arising in condensed matter physics that are described by continuum variational principles. The Landau-Lifshitz-Gilbert equation and the Gross-Pitaevskii (GP) equation are two paradigms for the models we addressed. Each problem involves topological information, as order parameters tend (by a penalty) to take values into a nontrivial manifold (for instance the unit sphere). The models are also multiscale and can be analyzed for large or small parameter regimes by means of asymptotic analysis. The formation of singularities in certain limits is due either to topological constraints or to the failure of some critical functional embeddings, and there is substantial overlap in some of the analytic difficulties (although magnetic and superconducting materials obey different dynamical laws).

Our scientific program was composed by three major parts:

(A) Pattern formation in micromagnetics. Micromagnetics, a continuum model for the behavior of ferromagnetic materials introduced by Landau-Lifshitz in the 30' and developed by Brown in the 60', can be seen as a typical example of multiscale problems. Indeed, ferromagnetic materials exhibit complex microstructures such as magnetic domains, domain walls and vortices. We studied the formation of such structures in order to predict the morphology and to determine the properties of patterns at different scales. These questions are of vital importance for a number of key technological applications.

(B) Dynamics of micromagnetic singularities. The fundamental dynamic law in ferromagnetism, given by the Landau-Lifshitz-Gilbert (LLG) equation, is a combination of precession and relaxation dynamics (thus neither a hamiltonian system nor a gradient flow). The balance of energetic stray-field forces and dynamic gyro-forces becomes singular in certain regimes. The resulting effects include complex oscillatory phenomena such as spin waves and resonances. Important ingredients for the evolution of micromagnetic patterns within a complex energy landscape, are effective motion laws for micromagnetic singularities that we investigated in specific situations.

(C) Vortex and traveling waves in the Gross-Pitaevskii equation. The Gross-Pitaevskii equation occurs in many areas in Physics from superfluidity and Bose-Einstein condensates to nonlinear optics. Although it was intended to be a simple model and was extensively studied in the last three decades, it is still far from being well understood. The balance between nonlinearity and dispersion gives traveling waves, while the nonzero condition at infinity and the penalty constraint in the energy generate vortices. It can be approximated by Euler's equations in some regime, or by the Kadomtsev-Petviashvili I equation in another regime.

3 Scientific approach

In this project, we developed new mathematical tools for the analysis of Gross-Pitaevskii and Landau-Lifshitz equations. In the mathematical models driven by these two equations, the interplay of only a very small number of basic parameters creates a wealth of phenomena, including ones that are experimentally and numerically largely inaccessible due to their multiscale nature. With the help of mathematical analysis, we rigorously derived results that explain the physical observations by relating them to the few basic effects that are the foundation of the model - thereby seconding (or disproving) its validity. In particular, to understand micromagnetics, we tested two new approaches:

(a) identifying the scaling law of the minimum energy, and the character of magnetization patterns that achieve it;

(b) identifying simpler models, valid in appropriate regimes, whose behavior is easier to understand or simulate (for example, the reduced model derived for asymmetric domain walls).

These new approaches are both based on asymptotic analysis (such as the Gamma-convergence method). We were able to take advantage of the presence of small nondimensional ratios, for example exchange length or film thickness divided by diameter of transversal section. For the asymptotic analysis, we assumed that some or all of these ratios tend to zero, with specified relations between them. The existence of more than one nondimensional parameter makes the problem rich, due to the presence of several distinct regimes. Previous research had established a firm mathematical foundation for many micromagnetic or superconducting phenomena, including the formation and dynamics of domain walls in magnetic thin-films, vortices / vortex lines for superconductors or dark solitons in optics. By exporting modern techniques from the calculus of variations and partial differential equations (PDEs), we were able to investigate some of the many subtle features of the theory remaining unelucidated such as the interaction energy between the Néel walls and their effective dynamic equation, the structure of asymmetric domain walls or the interaction between different modes in the Gross-Pitaevskii equation. We highlight that the mathematical methods we developed here have also their own interest by their richness, the variety of tools and the links they weave between different domains: analysis, PDEs and geometry.

4 Results

We summarize now our results following the items (A),(B),(C) presented in Section 2.

(A) Pattern formation in micromagnetics. It is an intriguing question how the simple, yet subtle, micromagnetic energy functional describes a rich variety of patterns on very different scales. Our aim was to develop mathematical methods to analyse this multiscale problem and characterize the formation of domain walls.

Symmetric domain walls. A first example is the (symmetric) Néel wall, a smooth transition layer characterized by a one-dimensional in-plane rotation connecting two (opposite) directions of the magnetization. Ignat-Moser [38, 40] succeeded to solve an open problem in the physical literature concerning the interaction energy between the Néel walls; more precisely, we computed the renormalized energy that governs the position of these walls. In two other articles (Ignat-Moser [34, 21]) we studied composite Néel walls that have a prescribed winding number; we succeeded to fully describe the diagram of existence / non-existence of such minimizing composite Néel walls in terms of the applied magnetic field and the prescribed winding number. We also mention here the papers Ignat-Monteil [14, 12] where we developed a general calibration/entropy method in order to prove the symmetry of domain walls (such as Bloch walls). More precisely, we determined a large class of anisotropies for which calibrations exist, thus yielding the 1D structure of these domain walls. Similar symmetry results can be given in the context of a phase-field-crystal model (see Ignat-Zorgati [16]).

Interior vortices and boundary vortices. Next to Néel walls, interior and boundary vortices play an important role as topological point defects of the magnetization. Recent progress in the study of boundary vortices has been done by Ignat-Kurzke [5] in a thin film regime where boundary vortices are energetically less expensive than interior vortices. More precisely, we developed a theory for the "global Jacobian" and the Gamma-convergence method in order to prove that the micromagnetic energy and the vorticity measure concentrate at the boundary and to determine the interaction energy between the boundary vortices that governs their optimal position. An important ingredient here is based on a global estimate for the modulus of solutions to a 2D Ginzburg-Landau equation proved by Ignat-Lamy-Kurzke [7]. In a recent work, Ignat-Jerrard [33, 4] studied a thin film regime of magnetic shells where vortices appear naturally on the limiting 2D surface. We determined the renormalized energy between vortex points as a Gamma-limit (at the second order); the interaction energy governing the optimal location of vortices depends on the Gauss curvature of the surface as well as on a quantized flux. The coupling between flux quantization constraints and vorticity, and its impact on the renormalized energy, are new phenomena in the theory of Ginzburg-Landau type models. We also mention here the works of Ignat-Nguyen-Slastikov-Zarnescu [44, 39, 28, 15, 23, 8] on vortex defects in nematic liquid crystals and superconductors, many features of these defects (stability, symmetry, uniqueness results) being similar to micromagnetic vortices. In the article [10], Goldman-Merlet-Millot studied defect patterns in a variational problem that is a combination of the complex Ginzburg-Landau functional and the Mumford-Shah functional allowing for line discontinuities in the orientation of the order parameter. They proved that minimizers of such functional develop finitely many point singularities of vortex type. In addition, those vortices are connected by line singularities to form clusters of entire topological degree, and each cluster is given by a Steiner tree connecting its own vortices. A phase field regularization of the classical 2D Steiner problem was studied by Bonnivard-Lemenant-Millot [24] which deals with the asymptotic behavior of minimizers of a Ginzburg-Landau type functional. A consistency result shows that level sets of minimizers converge to solutions of the Steiner problem in the small parameter limit. Let us mention that we also developed a more general theory on the structure and properties of maps with values into manifolds that may satisfy divergence or curl constraints: the vortex structure in the eikonal equation (see De Lellis-Ignat [43], Bochard-Ignat [30]) as well as the lifting structure for BV maps with values into projective spaces (see Ignat-Lamy [22]).

Asymmetric domain walls. A puzzling question concerns the cross-over from symmetric to asymmetric domain walls. Döring-Ignat [37] succeeded to analyse the bifurcation phenomenon in a regime where a symmetry breaking occurs and the (symmetric) Néel wall loses stability. The derived reduced model allows to capture asymmetric domain walls including their extended tails (which were previously inaccessible to brute-force numerical simulation). In the paper Ignat-Otto [18], we studied another asymmetric domain wall, called the magnetization ripple that is a microstructure formed by the magnetization in a thin film ferromagnet. It is triggered by the random orientation of the grains in the polycrystalline material. We developed a small-data well-posedness theory, taking inspiration from the recent rough-path approach to singular stochastic PDEs.

Singularities in nonlocal variationnal problems. In the articles [17, 27, 6, 11], Millot and coauthors have studied nonlocal fractional versions of the Ginzburg-Landau equation and the harmonic map system involving the fractional Laplacian. A main objective was to study the influence of such type of operator for the singularities induced by topological constraints. In [17], it is shown the convergence of solutions in the small (Ginzburg-Landau) parameter limit towards stationary nonlocal minimal surfaces; a regularity result for such nonlocal minimal surfaces is also obtained, and a connection with fractional harmonic maps is established. Articles [27, 6] address the regularity issue for fractional harmonic maps into spheres in the stationary or minimizing setting. In each case, a partial regularity result is obtained showing smoothness away from a closed singular set whose Hausdorff dimension is estimated in an optimal way. In the article [11], a classification result for all possible singularities is established in 2D for a critical exponent, a case of specific interest in the study of boundary singularities arising in micromagnetics or liquid crystals models.

(B) Dynamics of micromagnetic singularities.

Traveling waves in the Landau-Lifshitz-Gilbert equation. In the article Côte-Ignat [1], we describe precessing domain walls, and prove that they are asymptotically stable under an applied magnetic field which is sufficiently small. The method we developed also opens a line of research and provides tools to understand the interaction of several domain walls and the description of general magnetizations for large times. Periodic structures have been the object of the article Gustafson-Le Coz-Tsai [31]. The authors proved the stability of periodic waves in the framework of the nonlinear Schrödinger equation instead of the Landau-Lifshitz-Gilbert equation (which can be the object of further study using the tools developed in the framework of the Schrödinger equation). Also in the context of a nonlinear Schrödinger system, Delebecque-Le Coz-Weishaüpl [36] studied numerically and theoretically solitary waves. This study is continued in a work in progress by De Bièvre-Genoud-Le Coz-Rota Nodari on the Manakov system and its stationary waves. We mention the work Le Coz-Martel-Raphaël [42] where the authors constructed a blowing up solution for a nonlinear Schrödinger equation with double power nonlinearity. This solution has the peculiarity to be a blowing up solution at minimal mass whose speed does not correspond to any of the blowup speeds that were known before.

Vortex dynamics. The evolution of Ginzburg-Landau vortices was studied by Côte-Côte [35] and is given by a weak form of the mean curvature flow. For this, we extended the articles by Bethuel-Orlandi-Smets, the main contribution is to work in a physically relevant framework (i.e., locally finite energy density). We mention that the evolution of boundary vortices under the flow of the Landau-Lifshitz-Gilbert equation is a work in progress by Ignat-Kurzke.

(C) Vortex and traveling waves in the Gross-Pitaevskii equation.

Vorticity tubes. The evolution of vorticity tubes for the Gross-Pitaevskii equation (in its 3D axisymmetric version) is studied by Jerrard-Smets [26]; more precisely, they give a mathematical proof of the leap-frogging phenomenon, i.e., an interaction phenomenon between vortex tubes which was foreseen already by Helmholtz' in 1857 (in the case of incompressible perfect fluids), but for which a mathematical confirmation was not yet available. The same two authors then started the study of the evolution of vorticity tubes under no symmetry assumption. Very recently, they proved the validity of the Klein-Majda-Damodaran model in the limit of small curvature tubes. This work (still in progress) seems to be the first truly 3D mathematical result regarding the evolution of vortex tubes. In the articles [20, 13], Gallay-Smets studied the evolution of vortex tubes in the context of classical incompressible non viscous fluids (3D Euler equation). Although this was not explicitly mentioned in the ANR proposal, it is a very natural extension of the framework for the Gross-Pitaevskii equation which will surely benefit from it. The situation is notably more complex than for the Gross-Pitaveskii equation, due to co-existence of several scales. The authors proved under general assumptions the 3D spectral stability of a straight columnar vortex. By doing so, they answered positively a classical problem dating back to Kelvin, and for which no progress had been made since the 1960s.

Stability of localized structures. In a Gross-Pitaevskii-Schrödinger system, if the Schrödinger component becomes highly concentrated, it is natural to approximate the system as a Gross-Pitaevskii (GP) equation perturbed by a Dirac mass. This equation has been the subject of an in depth investigation (Cauchy problem and stability of stationary waves) in the article of Ianni-Le Coz-Royer [32]. Also, Le Coz-Wu [29] established the stability of multi-solitons for the derivative nonlinear Schrödinger equation. This result is one of the rare results concerning stability of multi-solitons; it is due to the particular structure of the equation, which is at the same time critical and admits stable solitons. In the article Côte-Martel [25], we constructed excited multi-solitons for the nonlinear Klein-Gordon equation: this is the first result of existence for progressive waves based on excited states without any restriction like high relative speeds. The main point is that the linearized flow around these states is linearly unstable, and beyond this, little is known (to the contrary of the case of ground states). On a similar topic, we described completely 2-soliton solutions to the damped nonlinear Klein-Gordon equation in Côte-Martel-Yuan-Zhao [3]: construction, description of the profile for large times, and smoothness of the set of initial data leading to 2-solitons (essentially, a Lipschitz manifold of codimension 2).

Interaction between modes. In the preprints by Correia-Côte-Vega [9, 2], we studied a dispersive model for the formation and evolution of vortex filaments making a corner. We studied the self-similar solutions to the modified Korteweg-de Vries equation (which corresponds to the curvature of the filaments), and we found a (critical) functional framework in which the flow can be studied. We mention that Maris [41] developed a very general profile decomposition method and used it for a broad family of minimization problems. Although it was initially motivated by the study of special solutions of the (GP) and (LLG) equations, it has been applied to many other problems (including solitary waves for dispersive equations, optimal functions for the Gagliardo-Nirenberg inequalities, liquid crystals...). In a work in progress, Alhelou-Maris study traveling waves for a Gross-Pitaevskii-Schrödinger system describing the movement of an impurity in a Bose-Einstein condensate. They proved the existence of such solutions with large momentum, small mass and speed close to zero. This result holds in any space dimension greater or equal than two. No existence result was available before, except for the dimension 1.

5 Discussion

The objectives proposed in our project have advanced very well. Our results made a significant progress in the theoretical study of the Landau-Lifshitz equation and the Gross-Pitaevskii equation. We succeeded to prove several conjectures in the physical and mathematical literature (such as determining the energy interaction between Néel walls, determining the evolution of vortex tubes in the Gross-Pitaevskii equation, the existence of traveling waves with large momentum, small mass and speed close to zero for a Gross-Pitaevskii-Schrödinger system). We developed new mathematical techniques that will have a strong impact in the study of singular structures in PDEs and Calculus of Variations (such as the calibration / entropy method to prove the symmetry of domain walls, or the decomposition method to prove stability or uniqueness of symmetric solutions). We opened new perspectives in the analysis of domain walls in micromagnetics. For example, we started the study of the structure of asymmetric domain walls in [37]; the next step consists in analyzing the bifurcation phenomenon for asymmetric type walls when a change of topological degree creates a new symmetry breaking (i.e., a vortex defect becomes favored by the system). This question will rely on the following new-insight issues:

1. the study of harmonic unimodular maps satisfying a divergence constraint and a prescribed degree;
2. a concentration-compactness principle for sign-changing functions.

Also, the study of vortices at the surface of thin magnetic shells in [4] highlighted new phenomena in the theory of Ginzburg-Landau type models: the coupling between flux quantization constraints and vorticity, and its impact on the renormalized energy. Our project was very ambitious, some of the many questions we raised in the project are still work in progress. For example, the dynamics of Néel walls for which the analysis we developed in [38, 21] is fundamental, or the dynamics of boundary vortices for the Landau-Lifshitz-Gilbert equation (for which the study at the stationary level of boundary vortices we did in [5] is essential). Also, we succeeded to describe precessing domain walls in [1], and proved that they are asymptotically stable under an applied magnetic field which is sufficiently small; the method we developed opens a new line of research and provides tools to understand the interaction of several domain walls and the description of the magnetization for large times.

6 Conclusion

We believe that our project was a truly success. We succeeded to make a significant progress in the analysis of topological singular phenomena in the Landau-Lifshitz equation and the Gross-Pitaevskii equation. We developed new mathematical methods that enabled us to prove important open questions in the physical and mathematical literature (as explained in the above sections). More than that, the development of these new tools and methods in nonlinear analysis can be used for a much larger class of problems than we expected in the original proposal. Therefore, they open new perspectives and will have a strong impact in the study of singular phenomena in PDEs and Calculus of Variations. Our project was very productive: 45 articles and reports-proceedings. Our papers are published in top journals in PDEs and Calculus of Variations (Annals of PDEs, ARMA, CalcVarPDEs, AnnIHP, Analysis &PDEs, CMP...) as well as top journals in Pure and Applied Mathematics (CPAM, AnnSciENS, Adv. Math, JMPA...). We were invited speakers at more than 45 national and international conferences to present our results. We also invited international experts as guests in our departments for several research visits or for the several conferences we organised in Toulouse; those scientific discussions turned in well established collaborations today. Also, during the project, 3 members of the project defended their habilitation HDR, 2 members became full professors, and 2 members became junior members at Institut Universitaire de France (IUF) with two research projects very related to our ANR project. We also mention that 6 of our students obtained their PhD degree during the project by working on related topics.

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