

Complex analysis and noncommutative functions
Workshop 3—7 October 2016, Toulouse
Titles and abstracts

Hari Bercovici (Indiana University): *Outliers for polynomial functions of random matrices.*

Abstract: We show how the matrix subordination functions can be used to determine the outlying eigenvalues of $p(A(n), B(n))$, where $A(n)$ and $B(n)$ are independent, unitarily invariant selfadjoint matrices, p is a selfadjoint polynomial, and the eigenvalues of $A(n)$ and $B(n)$ are distributed according to some limit measures and $A(n)$ has a fixed number of fixed eigenvalues outside the support of the corresponding limit measure.

(Wednesday, 10:15–11:15)

Yoann Dabrowski (Institut Camille Jordan): *Applications of non-commutative convex functions to free probability.*

Abstract: I will start by explaining several notions of non-commutative convex functions and give examples in each case. I will then survey two applications to free Gibbs states with potential V given by such non-commutative convex functions in emphasizing the use of convexity in each case. The first application will give free transport maps and is a joint work with Alice Guionnet and Dima Shlyakhtenko. The second application proves equality of microstates and non-microstates free entropy for such Gibbs states with convex potential.

(Thursday, 10:15–11:15)

Kenneth R. Davidson (University of Waterloo): *Choquet order and hyperrigidity for function systems*

Abstract: The Choquet order on measures is used to establish that states on a function system always have a representing measure supported on the set of extreme points of the state space (in a technical sense). We introduce a new operator-theoretic order on measures, and prove that it is equivalent to the Choquet order. This leads to some improvements in the classical theory, but more importantly it leads to some new operator-theoretic consequences. In particular, we establish Arveson's hyperrigidity conjecture for function systems. This yields a significant strengthening of the classical approximation theorems of Korovkin and Šaškin. This is joint work with Matthew Kennedy.

(Tuesday, 10:15–11:15)

William J. Helton (University of California San Diego): *Bianalytic maps in free variables*

Abstract: The talk concerns inequalities for functions having matrix variables. The functions are typically (noncommutative) polynomials or rational functions. A focus of much attention is the inequalities corresponding to convexity which in turn is bound closely to Linear Matrix Inequalities, LMIs.

Engineering systems problems seldom produce an LMI directly and depend on a change of variables to produce convexity. This talk concerns analytic changes of noncommutative variables to convert one convex set to another. This produces a wide range of subsidiary problems which need to be solved. Most of the work is done jointly by Meric Augat, J. William Helton, Igor Klep and Scott A. McCullough.

(Tuesday, 11:45–12:45)

Dmitry Kaliuzhnyi-Verbovetskyi (Drexel University): *Integrability of free noncommutative functions.*

Abstract: In our recent book "Foundations of Free Noncommutative Function Theory" with Victor Vinnikov, we formulated some necessary conditions for the integrability of a (free) nc function of order 1, i.e., for such a nc function to be the image of a nc function of order 0 under the right difference-differential operator. It turns out that under very general assumptions these conditions are also sufficient. I will present this result with a sketch of its proof. This is a joint work with Leonard Stevenson and Victor Vinnikov.

(Tuesday, 14:30–15:30)

Tobias Mai (Universität des Saarlandes): *Regularity questions for non-commutative distributions in free probability*

Abstract: During the last years, questions around the regularity of non-commutative distributions in the context free probability theory have attracted a lot of attention and some remarkable progress has been made in several directions. In contrast to classical probability, a measure theoretic description of non-commutative distributions is limited typically to the single-variable case. Correspondingly, the familiar notions of regularity, such as absence of atoms, absolute continuity, or smoothness of its density, do not apply directly. In the free world, there are essentially two alternative concepts for measuring the regularity of non-commutative distributions: firstly, by conditions based on quantities that arise from Voiculescu's groundbreaking work on analogues of entropy and Fisher's information measure in free probability theory; secondly, by evaluation of suitable "test functions" in the corresponding non-commutative random variables, which leads back to the single-variable case and hence to distributions whose regularity can be checked in the usual way. It is therefore a natural question whether and how these two approaches are related. Indeed, much work has been done in order to establish that regularity of the first kind implies regularity of the second kind. In my talk, I will report on some of these attempts, focusing on the joint work with R. Speicher and M. Weber, where non-commutative polynomials were used as test functions. Furthermore, I will discuss some possible refinements and extensions of this method, highlighting in particular the case of non-commutative rational functions, which is work in progress with F. Leid.

(Monday, 14:15–15:15)

Ion Nechita (TU München & CNRS): *Quantum channels, groups, and non-commutative iterative scaling algorithms*

Abstract: Motivated by two results in quantum information theory and quantum permutation groups, we introduce a (nother) non-commutative generalization of the Sinkhorn-Knopp iterative scaling algorithm. We discuss its convergence and compare it to Gurvits' version [J. of Comp. and Sys. Sci. 69 (2004)]. On the way, we discuss quantum Latin squares, as well as some open problems.

(Thursday, 11:45–12:45)

Mihai V. Popa (University of Texas at San Antonio): *TBA*

Abstract: TBA

(Monday, 11:45–12:45)

Eli Shamovich (Technion): *Multipliers, Reproducing Kernels and Varieties in the Free Ball*

Abstract: In this talk I will discuss the algebra of multipliers on the noncommutative H^2 of the unit ball, namely the noncommutative set of d -tuples of matrices (A_1, \dots, A_d) , satisfying $\sum_{k=1}^d A_k A_k^* \leq I$. This algebra was considered by many including Bunce, Popescu

and Davidson and Pitts. We will consider quotients of this algebra and the corresponding noncommutative varieties in the ball. We will discuss the isomorphism question between noncommutative varieties in the ball and completely isometric isomorphisms between the corresponding algebras of multipliers.

(Friday, 9:30–10:30)

In addition, a series of 5 lectures on noncommutative functions is offered by Prof. **Victor Vinnikov**. The summary can be found at <http://www.math.univ-toulouse.fr/%7Evfeuvrie/CIMI/download/Vinnikov-summary.pdf>