Journées du GdR ”Analyse Fonctionnelle et Harmonique et Probabilités”

Toulouse, October 10 to 12, 2016

1 About our sponsors

1.1 GDR AFHP

The GdR “Analyse Fonctionnelle, Harmonique et Probabilités” is a research network (“Groupement de Recherche”) funded by CNRS. It is a continuation of the GdR “Analyse Fonctionnelle, Harmonique et Applications”, created in 2000 and coordinated by Jean Esterle, then El-Maati Ouhabaz, Catalin Badea and Frédéric Bayart, in this order. Its current coordinator is Gilles Lancien.

Its purpose is to federate the various French research teams working on functional or harmonic analysis, and interacting subfields of probability.

More details can be found at http://gdrafhp.math.cnrs.fr/

1.2 CIMI

CIMI stands for “Centre International de Mathématiques et Informatique de Toulouse”, or International Center of Mathematics and Computer Science in Toulouse. It is one of the Excellence Laboratory chosen by the ANR (Agence Nationale pour la Recherche) for the period 2012-2020.

CIMI brings together the teams of the Institut de Mathématiques de Toulouse (IMT) as well as the teams of the Institut de Recherche en Informatique de Toulouse (IRIT).

Its programs include actions towards attractiveness, enabling a proactive strategy of high-level recruitments of confirmed and junior scientists and of outstanding students (French and foreigners). These include excellence chairs for long-term visitors and visiting chairs supported by partners from the industry on specific projects. Attractiveness is further enhanced with thematic trimesters organized within CIMI on specific topics including courses, seminars and workshops.

More details can be found at http://www.cimi.univ-toulouse.fr/en/what-cimi

This conference is part of a thematic semester in Analysis supported by CIMI.

More details can be found at http://www.cimi.univ-toulouse.fr/analysis/en

2 Schedule

Monday, October 10

11h30: Registration

12h30: Lunch

14h00: Welcoming remarks	
14h10: Xavier Tolsa	
amphithéâtre Schwartz	*Salle de conférence MIP*
15h00: Stéphane Charpentier	15h00: Cédric Arhancet
Coffee break	
16h: Oleg Szehr	16h: Quentin Menet
16h30: Frédéric Bernicot	

Tuesday, October 11

9h20: Emmanuel Breuillard	
amphithéâtre Schwartz	*Salle de conférence MIP*
10h10: Pierre Youssef	10h10: Rachid Zarouf
Coffee break	
11h: Omer Friedland	11h: Maria Trybula
11h30: Cécilia Lancien	11h30: Eric Amar
12h: Eugenia Malinnikova	

13h: Lunch

14h20: Alexei Poltoratski	
amphithéâtre Schwartz	*Salle de conférence MIP*
15h10: Hélène Bommier	15h10: Shahaf Nitzan
Coffee break	
16h: Daniel Seco	16h: Greta Marino
16h30: Florent Baudier	

20h: Dinner party at "Le Genty Magre"

3 rue Genty Magre, Toulouse (http://www.legentymagre.com/)

Wednesday, October 12

9h20: Joseph Lehec	
amphithéâtre Schwartz	*Salle de conférence MIP*
10h10: Ignacio Vergara	10h10: Elizabeth Strouse
Coffee break	
11h: Hubert Klaja	11h: Aingeru Fernandez-Bertolin
11h30: Jean Esterle	
12h: John E. McCarthy	

13h: Lunch

3 Abstracts of the long talks

Florent Baudier (Texas A&M) "Connections between Banach space geometry and the geometry of graphs":

It is by now a well-established fact that many geometric properties of Banach spaces are plainly connected to the geometry of certain families of graphs. Such explicit connections have had a fundamental transformative impact in theoretical computer science in the mid 90's, in particular in the design of approximation algorithms. Banach space geometry played a crucial role in this development. Indeed, some of the most important techniques and concepts used by theoretical computer scientists arose, more or less directly, from the investigation by Banach space geometers of the Ribe program. The Ribe program is concerned with metric characterizations of local properties of Banach spaces. These characterizations typically involve the geometry of some locally finite graphs. I will discuss at length the Ribe program and an emerging asymptotic version of it, the Kalton program. Discrepancies and analogies between these two programs, and some recent results will be emphasized. A significant part of the talk will be expository to accommodate an audience with broad and diverse interests.

Frédéric Bernicot (Nantes) "A bilinear principle of orthogonality":

We will describe several bilinear versions of Rubio de Francia's inequalities. These inequalities show that the square function, associated with disjoint projections in frequency, is bounded on L^p for $2 \leq p < \infty$. They describe a kind of L^p orthogonality of these Fourier projections. The L^2 estimate is a simple and direct consequence of Plancherel equality. In the bilinear setting, we can also consider square functions, associated with bilinear Fourier projections on disjoint squares. We will then give several results for the boundedness of such bilinear functionals. The bilinear question is far more difficult than the linear problem since there is no easy first estimates (as the L^2 one in the linear theory). The proofs relies on a very deep time-frequency decomposition / analysis, in order to encode the bilinear orthogonality. Part of this work is joint with Cristina Benea.

Emmanuel Breuillard (Münster) "About Bernouilli convolutions"

A Bernouilli convolution with parameter λ is the probability law on $\mathbb{R}$ of the of the sum of the series with terms $\pm\lambda^n$, where the $\pm$ are independent random variables taking the values 1 or -1 with probability $1/2$. The regularity of those measures with respect to the Lebesgue measure was studied by Erdős in the 1940s, but remains mysterious. The values of λ for which the measure has fractal dimension strictly less than 1 are exceptional. Following on Hochman's recent work, which proved that the exceptional values are of Hausdorff dimension 0, we show, in a joint work with P. Varju, that those values enjoy remarkable diophantine approximation properties. In particular, we reduce the problem to the case where the parameters λ are algebraic numbers. The proofs rely on entropy and introduce additive combinatorics arguments.

Joseph Lehec (Paris 9) "Borell type formulas and functional inequalities":

I will review some recent developments around Borell's stochastic formula for the Laplace transform of a function with respect to the Gaussian measure and its application to functional inequalities.

Eugenia Malinnikova (NTNU, Tronheim) "Uncertainty principles in various function spaces and irregular sampling":

The Heisenberg uncertainty principle implies that a well-localized function with unit norm in L^2 has large norm in the Sobolev space $W^{1,2}$. We will discuss generalizations of this idea by Strichartz and then present new results on localization in Besov spaces.

The talk is based on a joint work with Philippe Jaming.

John E. McCarthy (St Louis) "Matrix Monotone Functions of Several Variables":

In [2], K. Löwner characterized functions $f : [-1, 1] \to \mathbb{R}$ that were matrix monotone, which means that if S and T are self-adjoint contractive matrices and $S \leq T$, then $f(S) \leq f(T)$. In [1] we tried to characterize functions $f : [-1, 1]^d \to \mathbb{R}$ that satisfied $f(S) \leq f(T)$ whenever S and T are d-tuples of commuting self-adjoint contractions that satisfy $S_r \leq T_r$ for each r. We found a necessary condition, that is sufficient if $d = 2$ and f is rational. Whether it is always sufficient is unknown, and depends on whether local monotonicity implies global monotonicity.

[1] J. Agler, J.E. McCarthy, and N.J. Young. Operator monotone functions and Löwner functions of several variables. *Ann. of Math.*, 176(3):1783–1826, 2012.

[2] K. Löwner. Über monotone Matrixfunktionen. *Math. Z.*, 38:177–216, 1934.

Alexei Poltoratski (Texas A&M) "Gap theorems in mixed spectral problems":

A mixed spectral problem for Schroedinger equations asks if an operator can be recovered uniquely from partial information on the potential and spectrum. Every problem of this type quickly translates into a problem on uniqueness of holomorphic functions satisfying certain conditions. In my talk I will discuss the translation mechanism and show how recent results in complex and harmonic analysis give new solutions for mixed spectral problems for Schroedinger equations and Dirac systems.

Xavier Tolsa (Barcelona) "Riesz transforms, square functions, and rectifiability":

The geometric characterization of measures μ such that the associated Riesz transform of codimension 1 is bounded in $L^2(\mu)$ is a difficult problem with applications to other questions, such as the study of harmonic measure and the Lipschitz harmonic capacity. When μ is Ahlfors-David regular, by the solution of the David-Semmes problem (in codimension 1) by Nazarov, Volberg and myself, it turns out that the associated Riesz transform of codimension 1 is bounded in $L^2(\mu)$ if and only if μ is uniformly rectifiable. For more general measures, this characterization is more delicate and, for the moment there is not a complete solution. In this talk I will review some partial results which involve the so called β-Wolff potential, or Jones-Wolff potential.

4 Abstracts of the short talks

Eric Amar (Bordeaux) "On the cyclicity of bounded holomorphic functions":

We study the cyclicity of bounded holomorphic functions in some Hilbert spaces of holomorphic functions in the polydisc of $\mathbb{C}^n$. This work generalises some results of O. El-Fallah, K. Kellay and K. Seip, the main ingredient still being a "small corona theorem".

This is a joint work with P. Thomas.

Cédric Arhancet (Besançon) "Projections, multipliers and decomposable operators on noncommutative L^p spaces":

A linear operator $T\colon L^p(\Omega) \to L^p(\Omega)$ is called decomposable (regular) if it is a linear combination of positive operators T_k, i.e. $T_k f \geq 0$ for any $f \geq 0$. In this talk, we investigate decomposable maps acting on noncommutative L^p-spaces. We describe the noncommutative analogue of the absolute value of a decomposable operator. We also obtain detailed information regarding several classes of decomposable multipliers by constructing some kind of projections. This includes Schur multipliers, Fourier multipliers on some classes of locally compact groups. The approximation of groups by discrete groups plays an important role in this work.

The talk is based on joint work with Christoph Kriegler.

Hélène Bommier (Aix-Marseille) "Little Hankel operators on a class of vector-valued Fock spaces":

For a separable Hilbert space $\mathcal{H}$, we consider the vector valued Fock space $F^2_{m,\alpha}(\mathcal{H})$ of those holomorphic functions $f : \mathbb{C}^d \longrightarrow \mathcal{H}$ which are square integrable with respect to the measure $e^{-\alpha|z|^{2m}}, m \geq 1, \alpha > 0$. I will present some properties of the space $F^2_\alpha(\mathcal{H})$ and some spectral properties of the little Hankel operator h_b, of symbol $b : \mathbb{C}^d \longrightarrow \mathcal{L}(\mathcal{H})$, defined on $F^2_\alpha(\mathcal{H})$.

Stéphane Charpentier (Aix-Marseille) "Small Bergman-Orlicz spaces, growth spaces, and their composition operators":

Bergman-Orlicz spaces A^ψ and Hardy-Orlicz H^ψ spaces are natural generalizations of classical Bergman spaces A^p and Hardy spaces H^p. Their introduction makes sense in particular when one studies the boundedness or the compactness of composition operators on $A^p(\mathbb{B}_N)$ and Hardy spaces $H^p(\mathbb{B}_N)$ of the unit ball $\mathbb{B}_N$ of $\mathbb{C}^N$, $N \geq 1$. For instance, when $N > 1$, it is known that not every composition operator is bounded on $A^p(\mathbb{B}_N)$ and $H^p(\mathbb{B}_N)$ and it is a natural problem to find "reasonable" examples of Banach spaces of holomorphic functions on which every composition operator is bounded.

In this talk, we will characterize the Bergman-Orlicz spaces A^ψ on which every composition operators is bounded, in terms of the growth of the Orlicz function ψ. This will be done together with the characterization of those spaces, among the latter ones, which coincide with some growth spaces of holomorphic functions. By passing we will see that the boundedness of any operator on these spaces is equivalent to an *a priori* stronger property which, in the setting of A^2, correspond to being Hilbert-Schmidt. We will also say some words about the compactness of composition operators on small Bergman-Orlicz spaces.

Jean Esterle (Bordeaux) "About holomorphic functional calculus on semigroups of bounded operators":

We will have a fresh look at holomorphic functional calculus on semigroups in the very general framework of semigroups which are weakly continuous with respect to an "Arveson pair" (X, X^*). We introduce the Arveson ideal I associated to the semigroup and the algebras $QM(I)$ (resp. $QMr(I)$) of quasimultipliers (resp. regular quasimultipliers) on I. The algebra $QMr(I)$ is an inductive limit of Banach algebras, and the functional calculus associated to bounded holomorphic functions on suitable open set U such that $-U$ contains the Arveson spectrum of the generator of the semigroups takes values in $QMr(I)$, and so may take some values which do not represent bounded operators, but this functional calculus works for bounded operators not necessarily bounded at the origin. The usual H^∞ functional calculus for sectorial operators can be formulated in this framework. This work makes use of the theory of regular quasimultipliers developed by the author in the Proceedings of the Conference on Banach algebras and Applications organized in 1981 at Long Beach.

Joint work with Isabelle Chalendar and Jonathan Partington.

Aingeru Fernández-Bertolin (Bordeaux) "Discrete Hardy Uncertainty Principle and Schrödinger evolutions":

In Mathematics there are different properties that relate the behavior of a function to the behavior of its Fourier transform. In this talk, we will focus on the Hardy Uncertainty Principles, which state that a function and its Fourier transform cannot have Gaussian decay simultaneously, and we will also consider dynamical interpretations of this principle in terms of solutions to the Schrödinger equation.

The aim of the talk is to review this theory and adapt it to a discrete setting, assuming that our space variable is not in $\mathbb{R}^n$, but it is a point of the lattice $\mathbb{Z}^n$. Here the role of the Gaussian (which is a fundamental function in the continuous case) is played by the modified Bessel function

Omer Friedland (Paris 6) "Approximating matrices and convex bodies through Kadison-Singer":

We exploit the recent solution of the Kadison-Singer problem to show that any $n \times m$ matrix A can be approximated in operator norm by a submatrix with a number of columns of order the stable rank of A. This improves on existing results by removing an extra logarithmic factor in the size of the extracted matrix. As a corollary, we recover the sparsification result of Batson, Spielman and Srivastava with equal weights. We develop a sort of tensorization technique which allows to deal with a special kind of constraint approximation motivated by problems in convex geometry. As an application, we show that any convex body in $\mathbb{R}^n$ is arbitrary close to another one having $O(n)$ contact points. This fills the gap left in the literature after the results of Rudelson and Srivastava and completely answers the problem. As a consequence of this, we show that the method developed by Guédon, Gordon and Meyer to establish the isomorphic Dvoretzky theorem yields to the best known result once we inject our improvement of Rudelson's theorem. This is a joint work with Pierre Youssef.

Hubert Klaja (Centrale Lille) "Spectral sets for the numerical radius":

A domain $\Omega \subset \mathbb{C}$ is a spectral set for T if $\sigma(T) \subset \Omega$ and if, for every rational function f with poles outside Ω, we have

$$\| f(T) \| \leq \sup\{| f(z) | : z \in \Omega\}.$$

In this talk, we will discuss a numerical radius analogue of this notion.

This is a joint work with Javad Mashreghi and Thomas Ransford.

Cécilia Lancien (Lyon 1) "Random correlation matrices: when are they with high probability classical or quantum?":

Two observers performing binary outcome measurements on subsystems of a global system may obtain more strongly correlated results when they have a shared entangled quantum state than when they only have shared randomness. This well-known phenomenon of Bell inequality violation can be precisely characterized mathematically. Indeed, being a classical or a quantum correlation matrix exactly corresponds to being in the unit ball of some tensor norms. In this talk, I will start with explaining all this in details. I will then look at the following problem: given a random matrix of size n, can one estimate the typical value of its "classical" and "quantum" norms, as n becomes large? For a wide class of random matrices, the answer is yes, and shows a separation between the two values. This result may be interpreted as follows: in a typical direction, the borders of the sets of classical and quantum correlations do not coincide, or else: in a typical direction, the sets dual to the sets of classical and quantum correlations have different widths.

Based on joint work with C. Gonzalez-Guillen, C. Palazuelos, I. Villanueva.

http://arxiv.org/abs/1607.04203

Greta Marino (Catania) "Existence and asymptotic behaviour of bounded solutions to a class of Swift-Hohenberg equations":

We study existence and nonexistence of global solutions to the Swift-Hohenberg ($q > 0$) and Extended Fisher-Kolmogorov ($q \leq 0$) equation

$$u'''' + qu'' + f(u) = 0 \tag{1}$$

with a general nonlinearity f having super-linear growth at infinity and satisfying $f(u)u \geq 0$. For the S-H equation we ensure existence of periodic solutions for all $q > 0$, previously known only for sufficiently large q. We then study their asymptotic behavior for $q \to 0^+$, when $f'(0) = 0$. Contrary to the $f'(0) > 0$ case, we prove that for any sequence $q_n \to 0^+$ there exist corresponding solutions u_n of (1) such that $\|u_n\|_\infty \to 0$. For the EFK ($q \leq 0$) equation we prove blow-up of local solutions under a general condition on the growth of f at infinity alone. We finally discuss some open problems.

This is a joint work with S. Mosconi.

Quentin Menet (Lens) "Periodic points at the service of hypercyclicity":

Let X be an infinite-dimensional separable Banach space and T a continuous and linear operator on X. If a vector x possesses a dense orbit under the action of T, we say that the vector x is hypercyclic fot T. On the other hand, a periodic point for T is a vector x such that $T^d x = x$ for some $d \geq 1$. It can seem a little bit surprising but the existence of periodic points can be useful for obtaining hypercyclic vectors. The goal of this talk will consist in showing how the existence of a dense set of periodic points satisfying some properties can be used for constructing hypercyclic vectors and other stronger notions of hypercyclic vectors such as U-frequently hypercyclic vectors and frequently hypercyclic vectors.

Shahaf Nitzan (Georgia Tech.) "Persistence as a spectral property":

A Gaussian stationary sequence is a random function $f : \mathbb{Z} \to \mathbb{R}$, for which any vector $(f(x_1), ..., f(x_n))$ has a centered multi-normal distribution and whose distribution is invariant to shifts. Persistence is the event of such a random function to remain positive on a long interval $[0, N]$. Estimating the probability of this event has important implications in engineering , physics, and probability. However, though active efforts to understand persistence were made in the last 50 years, until recently, only specific examples and very general bounds were obtained. In the last few years, a new point of view simplifies the study of persistence, namely - relating it to the spectral measure of the process. In this work we use this point of view to develop new spectral and analytical methods in order to study the persistence in cases where the spectral measure is 'small' near zero. This talk is based on Joint work with Naomi Feldheim and Ohad Feldheim.

Daniel Seco (Barcelona) "Extremal problems and approximants for cyclicity":

We study the structure of the zeros of optimal polynomial approximants to reciprocals of functions in Hilbert spaces of analytic functions in the unit disk. In many instances, we find the minimum possible modulus of occurring zeros via a nonlinear extremal problem associated with norms of Jacobi matrices. We examine global properties of these zeros and prove Jentzsch-type theorems describing where they accumulate. As a consequence, we obtain detailed information regarding zeros of reproducing kernels in weighted spaces of analytic functions.

Elizabeth Strouse (Bordeaux) The Szego limit theorems for Toeplitz operators describe a limiting relationship between the eigenvalues or determinants of the upper left hand corners of Toeplitz matrices and integrals involving the symbol of the Toeplitz operator. The $n \times n$ upper left hand corners is really the compression of the Toeplitz operator to the quotient of the Hardy space H^2 by the closed subspace $z^n H^2$. We look at the generalization of this theorem when these quotients are replaced by certain model spaces (replacing the z^n by sequences of inner functions).

Oleg Szehr (Aix-Marseille) One of the basic tasks in matrix analysis is to find a spectral estimate to the norm of a function of a matrix. Examples abound in various forms e.g. as bounds on condition numbers in numerical stability analysis, as convergence estimates in the theory of Markov chains or as spectral variation bounds in linear algebra. At the core of our investigation lies the observation that a specific functional calculus can be used to associate a class of operators to a respective function space. Such a calculus provides a link between the problem of finding a spectral estimate and a Nevanlinna-Pick interpolation problem in the associated function space, see [1]. To approach the interpolation problem we employ tools from the theory of Hilbert function spaces (e.g. Sarasons's commutant lifting theorem), which provides an interesting application of advanced harmonic analysis tools to matrix theory. In this talk a special emphasis will be put on spectral estimates for Hilbert space contractions. In this case the Nagy-Foias commutant-lifting approach to interpolation theory [2] relates spectral estimates to norms of so-called compressed Toeplitz operators. We contribute to the theory of such operators by providing explicit matrix representations and by computing norms of inverses and resolvents [3].

[1] N.K.Nikolski, Condition numbers of large matrices and analytic capacities

[2] Sz.Nagy, C. Foias, Commutants de certains operateurs

[3] O.Szehr, Eigenvalue estimates for the resolvent of a non-normal matrix

Maria Trybula (Poznan) "Hadamard multipliers on spaces of holomorphic functions"

We consider multipliers on the spaces of holomorphic functions in one variable. We prove representation theorem in terms of analytic functionals and in terms of holomorphic functions. We also consider the problem when the spaces of multipliers endowed with the strong topology are isomorphic (as locally convex spaces) with the spaces of analytic funtionals.

Ignacio Vergara (ENS Lyon) "The p-approximation property for simple Lie groups with finite center":

Let $1 < p < \infty$. We say that a locally compact group G has the p-approximation property (p-AP) if the constant function 1 can be approximated by a net of elements of the Figà-Talamanca–Herz algebra $A_p(G)$ in the weak* topology of the space of p-completely bounded multipliers $M_{p\text{-}cb}(G)$. When $p = 2$ this corresponds to the approximation property (AP) of Haagerup and Kraus. The result that I will present states that all connected simple Lie groups with finite center and real rank greater than 1 fail to have this property for all $1 < p < \infty$. This extends a result by Haagerup and de Laat, which was inspired by the work of Lafforgue and de la Salle on the group $\mathrm{SL}(3, \mathbb{R})$. The strategy of the proof is the same; it is sufficient to prove the result for the groups $\mathrm{SL}(3, \mathbb{R})$ and $\mathrm{Sp}(2, \mathbb{R})$, and then deduce the general case using some Lie group theory and some stability properties of the p-AP.

Pierre Youssef (Paris 7) "Trou spectral pour les graphes aléatoires uniformes dans le régime dense".

On note λ la deuxième plus grande valeur propre en valeur absolue d?un graphe aléatoire d-régulier sur n-sommets suivant le modèle uniforme. Friedman a montré la conjecture d'Alon qui affirmait que lorsque le degré d est une constante indépendante de n, alors $\lambda \leq 2\sqrt{d-1} + o(1)$ avec probabilité qui tend vers 1 avec n. Ceci signifie qu'il y a un écart avec la plus grande valeur propre qui est égale à d et montre que les graphes d-réguliers uniformes sont presque Ramanujan. Vu a conjecturé que cette borne reste valable pour tout $d \leq n/2$. Des avancées sur ce problème ont été réalisées par Broder, Frieze, Suen et Upfal qui ont montré que $\lambda \leq O(\sqrt{d})$ pour tout $d \leq \sqrt{n}$. Le régime de d a été étendu récemment à $d \leq n^{2/3}$ par Cook, Goldstein et Johnson. Nous complétons ces résultats en montrant que pour tout $\delta \in (0, 1)$, on a $\lambda \leq O(\sqrt{d})$ pour tout $n^{\delta} \leq d \leq n/2$. Ceci montre à constante près la conjecture faite par Vu.

Ce travail est en collaboration avec Konstantin Tikhomirov.

Rachid Zarouf (Marseille) "On the asymptotic behavior of Jacobi polynomials with varying parameters":

We study the large n behavior of Jacobi polynomials with varying parameters $P_n^{(an+\alpha,\beta)}(1 - 2\lambda^2)$ for $a > -1$ and $\lambda \in (0, 1)$. This appears to be a well-studied topic in the literature but some of the published results are unnecessarily complicated or incorrect. The purpose of this talk is to provide a simple and clear discussion and to point out some flaws in the existing literature. Our approach is based on a new representation for $P_n^{(an+\alpha,\beta)}(1 - 2\lambda^2)$ in terms of two integrals. The integrals' asymptotic behavior is studied using standard tools of asymptotic analysis: one is a Laplace integral and the other is treated via the method of stationary phase. We prove that if $a \in (\frac{2\lambda}{1-\lambda}, \infty)$ then $\lambda^{an} P_n^{(an+\alpha,\beta)}(1 - 2\lambda^2)$ exhibits exponential decay, and we derive exponential upper bounds in this region. If $a \in (\frac{-2\lambda}{1+\lambda}, \frac{2\lambda}{1-\lambda})$ then the decay of $\lambda^{an} P_n^{(an+\alpha,\beta)}(1 - 2\lambda^2)$ is $\mathcal{O}(n^{-1/2})$ and if $a \in \{\frac{-2\lambda}{1+\lambda}, \frac{2\lambda}{1-\lambda}\}$ then $\lambda^{an} P_n^{(an+\alpha,\beta)}(1 - 2\lambda^2)$ decays as $\mathcal{O}(n^{-1/3})$. Lastly we find that if $a \in (-1, \frac{-2\lambda}{1+\lambda})$ then $\lambda^{an} P_n^{(an+\alpha,\beta)}(1 - 2\lambda^2)$ decays exponentially iff $an + \alpha$ is an integer and increases exponentially iff it is not.

Our methods immediately yield the asymptotic expansion of the polynomials to any order.

[1] S. H. Izen, Refined estimates on the growth rate of Jacobi polynomials, J. Approx. Theory 144-1, 54-66 (2007).

[2] L.-C. Chen, M. E. H. Ismail, On asymptotics of Jacobi polynomials, SIAM J. Math. Anal. 22-5, 1442-1449 (1991).

[3] A. Erdélyi, Asymptotic representations of Fourier integrals and the method of stationary phase, J. Soc. Indust. Appl. Math., 3, 17-27 (1955).

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